WAVE BASED MODELING OF AIRBORNE FLANKING TRANSMISSION ACROSS SUSPENDED CEILINGS

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Abstract
In this paper, a wave based model is developed to assess the flanking sound transmission between two adjacent rooms through the plenum of suspended ceilings. The model accounts for the modal behaviour of the rooms, the plenum and the suspended ceilings. The plenum can either be empty or partly filled with an absorbent layer. Results are compared with a classical three-room model that assumes diffuse sound fields in both rooms and the plenum. At very low frequencies, the sound transmission is determined by the modal behaviour of the global system. At middle frequencies, the sound field in the plenum is two-dimensional. As a result, the three-room model overestimates the airborne flanking transmission in this frequency range. The transmission loss decreases at a cut-on frequency that is determined by the height of the plenum and above which the sound field in the plenum is three-dimensional.

Keywords: airborne flanking, suspended ceilings, wave based model.

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1 Introduction

In non-residential buildings, rooms are often separated by lightweight partition walls beneath a suspended ceiling. In this case, sound transmission through the plenum can form an important airborne flanking path. Because the dimensions of the plenum are relatively small, statistical models fail to predict the flanking sound transmission through suspended ceilings with good accuracy in a broad frequency range. Furthermore, testing of airborne flanking path transmission is cumbersome.
Hamme [1,2] developed a standardized laboratory measurement set-up to determine the performance of different suspended ceiling systems. It proved however difficult to interpret and compare measurement results because the sound transmission through suspended ceilings is complicated by the interaction between several factors like ceiling transmission, absorption, and plenum duct propagation losses. These factors are all influenced by both the ceiling material, the suspension system and the dimensions. In typical suspended ceiling constructions with modular plates, the overall sound transmission may be influenced by acoustical leakage that occurs between individual elements of the ceiling and the suspension system. The presence of air diffusers, grilles and lights can further reduce the performance. Halliwell and Quirt [3] investigated the possible improvement by using a stack of absorptive batts as a sound barrier above the partition wall in the plenum. Both laboratory and field

measurements showed that the batts can effectively block sound transmission through the plenum and the efficiency is increased by increasing the thickness of the batts.

Mariner [4] developed an analytical model based on one-dimensional wave propagation in ducts and assuming diffuse sound fields in the rooms. The sound transmission is estimated in function of the transmission and absorption coefficients of the suspended ceiling, the absorption of the plenum walls and ceiling, and the room lengths. Mechel [5,6] developed different models based on a modal superposition approach and compared results with transmission loss measurements for different suspended ceiling systems. While the modeling approach is similar to the wave based model developed in this paper, the models are less general due to the assumptions of symmetry and equal mode density distribution in the source room.

In this paper, it is the aim to develop a model for the airborne flanking sound transmission through suspended ceilings that allows to investigate the influence of different parameters independently across the broad building acoustics frequency range.

2 Wave based model

The wave based method (WBM) is a Trefftz-based deterministic prediction method for the steady-state dynamic analysis of coupled vibro-acoustic systems [7]. The field variables are approximated by a set of wave functions, which are exact solutions of the homogeneous part of the governing dynamic equations. The contribution of a certain function in a set is thus only determined by the boundary and continuity conditions. For problem geometries with a moderate complexity, as considered in this paper, the method has an enhanced computational efficiency compared to element-based approaches such as the finite element method.

2.1 Problem definition

The geometry of the considered problem is shown in Figure 1. Two adjacent rectangular rooms with a continuous suspended ceiling are separated by a rigid wall. In this way, direct sound transmission between the rooms is disregarded and only flanking through the plenum is taken into account. The side and back walls of the rooms and the plenum are assumed rigid. Source and receiving room have dimensions $L_x^{(1)} \times L_y^{(1)} \times L_z^{(1)}$ and $L_x^{(3)} \times L_y^{(3)} \times L_z^{(3)}$, respectively. The plenum has a height $L_z^{(2)}$ and can either be empty or (partly) filled with an absorbent material. The suspended ceiling consists of two plates (with dimensions $L_x^{(1)} \times L_y^{(1)}$ and $L_x^{(3)} \times L_y^{(3)}$) and it is assumed that there are no structural connections between the plates. Although structural flanking may occur in reality as it is common practice in field construction to build an expanse of ceiling and then erect partitions to the underside, the model allows for an exclusive assessment of the airborne flanking path. In this paper, the boundary conditions of the plates are assumed simply supported, although free or clamped boundary conditions are also integrated in the general model. A harmonic volume point source is placed in the source room at local coordinates (x_s, y_s, z_s). It is assumed that the rooms have equal width ($L_y^{(1)} = L_y^{(3)}$) and height ($L_z^{(1)} = L_z^{(3)}$) and that the plenum covers the whole floor area of the rooms ($L_x^{(2)} = L_x^{(1)} + L_x^{(3)}$ and $L_y^{(2)} = L_y^{(1)}$), but the method can easily be extended to cases where the dimensions of rooms and plenum differ.

It must be noted that the plenum above the suspended ceiling in a real building typically extends over more than two rooms in all directions, making the rigid side wall assumption of the plenum unrealistic. In laboratory experiments, absorptive lining is placed on the sidewalls of the plenum to simulate the large lateral extent of the plenum. This can significantly reduce the sound transmission through

plenums without absorption as reflections are mostly eleminated as it would be in a plenum of infinite extent [2].
To simplify some expressions, local coordinate systems (x',y',z') are used. For the source room and the plenum, the origin of the local axes is placed at the ceiling: $(x',y',z')=(x,y,z-L_z^{(1)})$. For the receiving room, the local coordinate system is defined with the origin of the axes in the top left corner: $(x',y',z')=(x-L_x^{(1)},y,z-L_z^{(3)})$. Also for the plates, local coordinates are introduced that correspond with the coordinates of the source and receiving room, respectively.

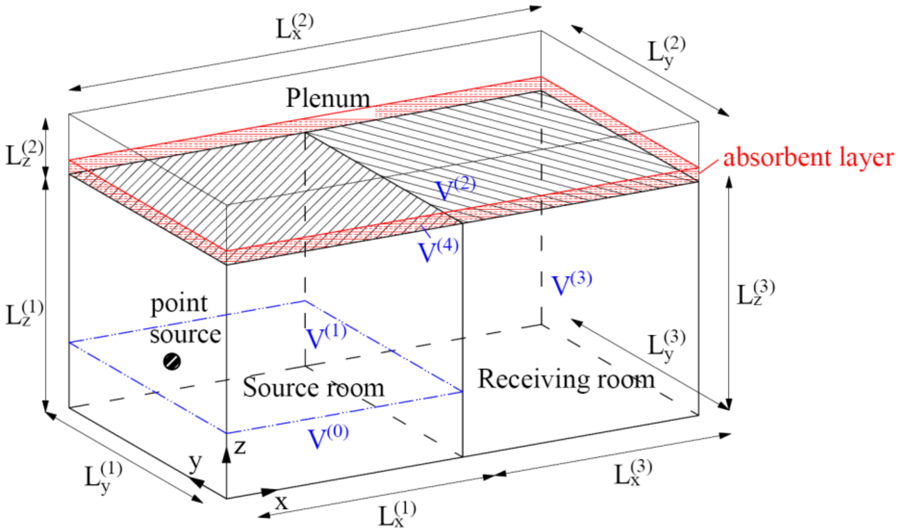


Figure 1 – Geometry of the wave based model: a suspended ceiling with or without cavity absorption above two rooms with rigid side and back walls.

2.1.1 Rooms and plenum

The source room is divided into two subdomains ($V^{(0)}$ and $V^{(1)}$) by a plane through the point source, parallel to the ceiling wall. The steady-state acoustical pressure $\underline{p}_a^{(i)}$ in each (sub)room $V^{(i)}$ ($i=0\ldots3$) is governed by the homogeneous Helmholtz equation:

$$\nabla^2 \underline{p}_a^{(i)}(x',y',z')+\underline{k}_a^2\,\underline{p}_a^{(i)}(x',y',z')=0\,. \tag{1}$$

$\underline{k}_a=\omega/c_a$ is the acoustic wave number in air, with ω the circular frequency and c_a the speed of sound in air. Uniform spatial damping is introduced by making the acoustic wavenumber complex [8].

2.1.2 Plenum absorption

To reduce the airborne flanking transmission across the plenum, it can be filled with a sound absorbing material. The addition of acoustical absorption to the cavity will reduce the amplitude of standing waves and will result in an increase in the transmission loss. In the wave based model, it is assumed that an absorbent material with thickness d is placed in the entire plenum. The plenum is divided in two subdomains, $V^{(2)}$ and $V^{(4)}$ (Figure 1). An equivalent fluid model is used for the cavity absorption. The pressure in the equivalent fluid ($V^{(4)}$) fulfils the Helmholtz equation with an adjusted wave number. Several theories can be used to calculate the wave number and complex density of a porous material with a motionless frame, like the theory of Biot-Johnson-Allard [9], or empirical models like that of Delany and Bazley [10], where only the ratio of frequency to airflow resistivity is required to predict the acoustic performance.

2.1.3 Plates

The plates are assumed homogeneous, isotropic and acoustically thin. The transverse displacement of the plates thus fulfils Kirchhoff's thin plate bending wave equation:

$$\underline{B}'_1 \nabla^4 \underline{w}_p^{(1)}(x',y') - m''_1 \omega^2 \underline{w}_p^{(1)}(x',y') = \underline{p}_a^{(1)}(x',y',0) - \underline{p}_a^{(2)}(x',y',0), \tag{2}$$

$$\underline{B}'_2 \nabla^4 \underline{w}_p^{(2)}(x',y') - m''_2 \omega^2 \underline{w}_p^{(2)}(x',y') = \underline{p}_a^{(3)}(x',y',0) - \underline{p}_a^{(2)}(x',y',0). \tag{3}$$

Here, $m_i'' = \rho_i h_i$ is the surface mass density and $\underline{B}'_i$ the bending stiffness of plate i:

$$\underline{B}'_i = \frac{E_i h_i^3 (1 + j\eta_i)}{12(1 - \nu_i^2)}, \tag{4}$$

with $j = \sqrt{-1}$. Plate i with thickness h_i has a density ρ_i, a Young's modulus E_i, a loss factor η_i and a Poisson's ratio ν_i. When plenum absorption is present, the pressures $\underline{p}_a^{(2)}(x',y',0)$ in the right-hand side of Eq. (2) and Eq. (3) should be replaced with $\underline{p}_a^{(4)}(x',y',0)$.

2.2 Field variable expansions

The acoustic pressures are approximated in terms of the following acoustic wave function expansion:

$$\underline{p}_a^{(i)}(x',y',z') = \sum_m \sum_n \left(e^{-j\underline{k}_{zmn}^{(i)} z'} \underline{P}_{mn}^{(i)} + e^{j\underline{k}_{zmn}^{(i)} z'} \underline{Q}_{mn}^{(i)} \right) \cos\left(\frac{m\pi}{L_x^{(i)}} x' \right) \cos\left(\frac{n\pi}{L_y^{(i)}} y' \right). \tag{5}$$

where

$$\underline{k}_{zmn}^{(i)} = \sqrt{ \left(\underline{k}_a^{(i)} \right)^2 - \left(\frac{m\pi}{L_x^{(i)}} \right)^2 - \left(\frac{n\pi}{L_y^{(i)}} \right)^2 }, \tag{6}$$

with $m,n = 0,1,2,\ldots$. The wave functions are exact solutions of the homogeneous Helmholtz equation. The time dependence $e^{j\omega t}$ has been omitted throughout this paper. Using Euler's equation, Eq. (5) leads to the following wave function expansion for the particle displacement in the z-direction:

$$\underline{w}_{za}^{(i)}(x',y',z') = \frac{-j}{\omega^2 \rho_a} \sum_m \sum_n \underline{k}_{zmn}^{(i)} \left(e^{-j\underline{k}_{zmn}^{(i)} z'} \underline{P}_{mn}^{(i)} - e^{j\underline{k}_{zmn}^{(i)} z'} \underline{Q}_{mn}^{(i)} \right) \cos\left(\frac{m\pi}{L_x^{(i)}} x' \right) \cos\left(\frac{n\pi}{L_y^{(i)}} y' \right), \tag{7}$$

with ρ_a the density of air.

Also for the transverse displacement of the plates, a field variable expansion is used:

$$\underline{w}_p^{(i)}(x',y') = \sum_p \sum_q \underline{A}_{pq}^{(i)} \sin\left(\frac{p\pi}{L_x^{(i)}} x' \right) \sin\left(\frac{q\pi}{L_y^{(i)}} y' \right), \tag{8}$$

with $p,q=1,2,3,\ldots$ These expansions satisfy a priori the simply supported boundary conditions.

2.3 Boundary and continuity conditions

The participation factors in the pressure expansions are determined by the boundary and continuity conditions. The proposed wave functions satisfy a priori the rigid side wall boundary conditions. In the source and the receiving room, a rigid floor is assumed,

$$\underline{w}_{za}^{(0)}(x',y',-L_z^{(1)})=0 \ , \tag{9}$$

$$\underline{w}_{za}^{(3)}(x',y',-L_z^{(3)})=0 \ . \tag{10}$$

At the source plane $z'=z_s$, continuity of pressure and particle velocity is imposed,

$$\underline{p}_a^{(0)}(x',y',z_s)=\underline{p}_a^{(1)}(x',y',z_s) \ , \tag{11}$$

$$j\omega\underline{w}_{za}^{(0)}(x',y',z_s)+\delta(x'-x_s,y'-y_s)=j\omega\underline{w}_{za}^{(1)}(x',y',z_s) \ . \tag{12}$$

In the plenum, following boundary condition holds when assuming a rigid ceiling,

$$\underline{w}_{za}^{(2)}(x',y',L_z^{(2)})=0 \ . \tag{13}$$

At the plate surfaces, continuity of transverse displacement is imposed,

$$\underline{w}_{za}^{(1)}(x',y',0)=\underline{w}_p^{(1)}(x',y')=\underline{w}_{za}^{(2)}(x',y',0) \ , \tag{14}$$

$$\underline{w}_{za}^{(2)}(x',y',0)=\underline{w}_p^{(2)}(x',y')=\underline{w}_{za}^{(3)}(x',y',0) \ . \tag{15}$$

When cavity absorption is present in the plenum, additinal continuity conditions have to imposed at the interface $z'=d$,

$$\underline{p}_a^{(2)}(x',y',d)=\underline{p}_a^{(4)}(x',y',d) \ , \tag{16}$$

$$\underline{w}_{za}^{(2)}(x',y',d)=\phi\underline{w}_{za}^{(4)}(x',y',d) \ , \tag{17}$$

with ϕ the porosity of the porous material. Furthermore, the continuity conditions (14) and (15) between the plate and the equivalent fluid in the plenum have to be adjusted to,

$$\underline{w}_p^{(1)}(x',y')=\phi\underline{w}_{za}^{(4)}(x',y',0) \ , \tag{18}$$

$$\underline{w}_p^{(2)}(x',y')=\phi\underline{w}_{za}^{(4)}(x',y',0) \ . \tag{19}$$

2.4 Construction of system matrices and post-processing

Because of the simple rectangular geometry, the factors $\underline{P}_{mn}^{(i)}$ and $\underline{Q}_{mn}^{(i)}$ can be eliminated analytically in function of the primary unknowns $\underline{A}_{pq}^{(i)}$ by use of the weighted residual formulations of the boundary and interface conditions (9)-(19). A weighted residual formulation of the equation of motion of the plates, i.e. Eq. (2)-(3), then results in a symmetric system of linear equations in the unknowns $\underline{A}_{pq}^{(i)}$. After the wave function contribution factors are determined, the transmission loss (TL) is determined by following measurement formula:

$$TL = L_{p1} - L_{p3} + 10\log\frac{S}{A_3} \ , \tag{21}$$

where L_{p1} and L_{p3} are the average sound pressure level in source and receiving room (which can be calculated analytically), S is the surface area of the common wall between source and receiving room and $A_3 = \dfrac{0.16V_3}{T_3}$ is the absorption area of the receiving room, with V_3 the volume and T_3 the reverberation time. Calculations are performed at 81 frequencies per one-third octave band up till 1250 Hz and 9 frequencies per one-third octave band up till 2000 Hz. The one-third octave band TL is determined from the band-averaged sound pressure levels.

3 Results

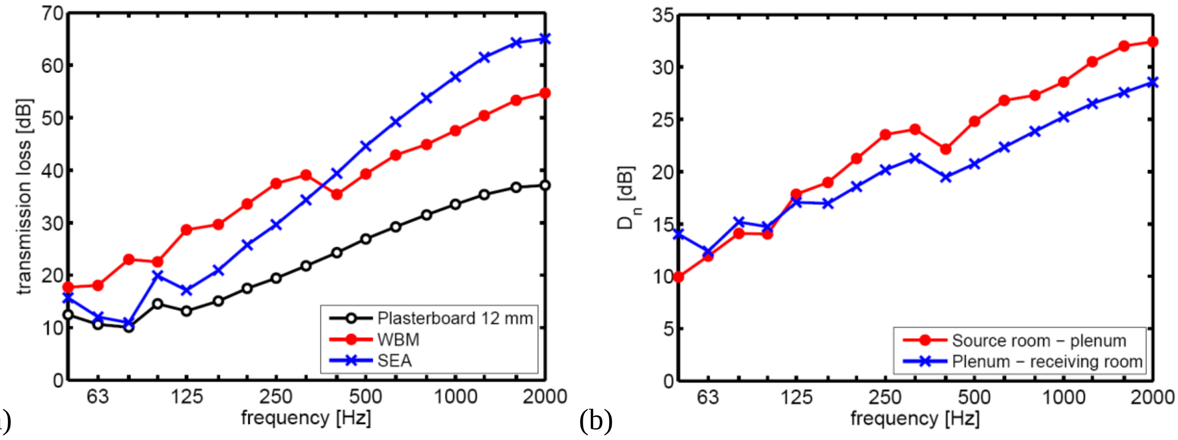
The wave based model presented in the previous section is used to predict the airborne flanking transmission across different suspended ceilings. First, the TL of a suspended ceiling with an empty plenum is investigated. Afterwards, the influence of cavity absorption and the plenum height is discussed. In the reference case, the source and receiving room have dimensions $L_x \times L_y \times L_z$ = 4.0 m x 3.5 m x 3.0 m and the plenum has a height L_z = 0.5 m. The ceiling is made of 12 mm plasterboard plates with material properties ρ = 1200 kg/m³, E = 3500 MPa, $\eta = 0.025$ and $\nu = 0.20$. The source is placed in the left bottom corner of the source room, $(x_s, y_s, z_s) = (0,0,-3)$ m.

3.1 Suspended ceiling without cavity absorption

In the first example, the plenum is empty and has a reverberation time of 1.0 s. Figure 2 shows the TL of the airborne flanking path across the suspended ceiling as calculated with the WBM. The WBM results are also compared with a statistical three-room model that assumes a diffuse sound field in the plenum:

$$TL = TL_1 + TL_2 + 10\log\frac{SA_2}{S_1S_2} \ , \tag{22}$$

where TL_1 and TL_2 are the transmission loss of the first and second ceiling plate, respectively; S_1 and S_2 are the surface area of the first and second ceiling plate, respectively; and A_2 is the



(a) (b)

Figure 2 – (a) TL of the airborne flanking path across a suspended ceiling without cavity absorption and (b) normalized sound pressure level differences between rooms and plenum.

equivalent absorption area of the plenum. The absorption area of the plenum is determined from the reverberation time using Sabine's formula. The TL of the ceiling plates is calculated with a wave based model for direct sound transmission [8] using the same source and receiving room but placed on top of each other. The TL of the ceiling plate is also shown in Figure 2a. Between 125 and 2000 Hz, it increases with frequency according to the mass law. Below 125 Hz, oscillations can be observed due to the modal behaviour of the rooms and the plate. Because the reverberation time of the plenum is assumed frequency independent, the SEA result for the flanking path follows the same trends. Below 125 Hz, modal fluctuations dominate the TL; at higher frequencies the TL increases with approximately 12 dB per octave. While the SEA model assumes that the normalized sound pressure level difference D_n between source room and plenum and between plenum and receiving room is the same, this is not the case in the WBM model (Figure 2b). Above 125 Hz, D_n between source room and plenum is systematically larger than D_n between plenum and receiving room, which is consistent with the findings of Mechel [6]. Three frequency regimes can be distinguished in the WBM results for the airborne flanking path (Figure 2a). The modal behaviour again determines the TL below 125 Hz. Between 125 Hz and 315 Hz, the WBM predicts a higher TL compared to the SEA model. Between 315 and 400 Hz, the TL drops resulting in lower TL values compared to the SEA results. These results can be understood by looking at the sound pressure level distributions in the rooms (Figure 3). At very low frequencies, the TL is determined by the global modal behaviour of the system. At 125 Hz for example (Figure 3a), the weak coupling between the room and plenum modes leads to a high TL. Above 125 Hz, the sound field in the source room becomes more and more diffuse. The sound field in the plenum stays two-dimensional, however, as the acoustic wavelength is larger than the plenum height (Figure 3b). The sound waves impinge upon the second ceiling plate at grazing angles of incidence. As a result, the sound field in the receiving room is also two-dimensional. Above the cut-on frequency of 343 Hz at which the plenum height equals half the acoustic wavelength, the sound field in the plenum is three-dimensional (Figures 3c and 3d).

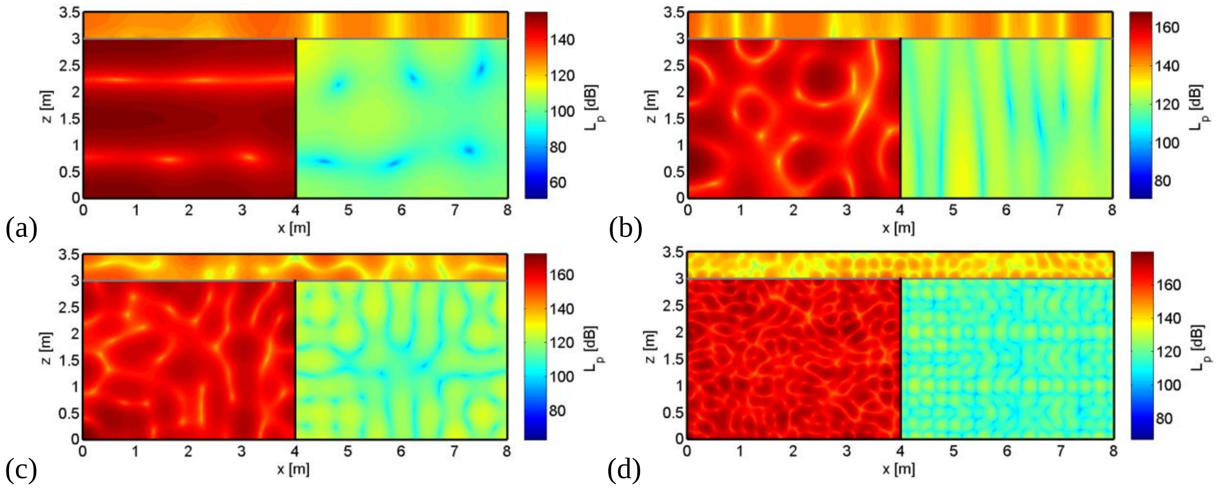


Figure 3 – Sound pressure level distribution in the plane y = 1 m at a frequency of (a) 125 Hz, (b) 300 Hz, (c) 400 Hz and (d) 1000 Hz for the suspended ceiling without cavity absorption.

3.2 Influence of cavity absorption

It is known that the sound transmission across the suspended ceiling can be reduced by placing absorption in the plenum. Here, it is investigated how the TL varies when a layer of mineral wool is placed on top of the suspended ceiling. The mineral wool (flow resistivity σ = 5000 Ns/m^4) is modeled as an equivalent fluid using the empirical relations of Delany-Bazley [10]. Figure 4 shows that an absorbing layer of 5 cm already significantly improves the TL from 125 Hz upwards. The improvement is the largest at higher frequencies. Increasing the thickness of the mineral wool to 15 cm will further enhance the sound insulation. The mineral wool suppresses the plenum modes and therefore the overall sound pressure level in the plenum is reduced (Figure 5). While L_p is relatively constant in the part of the plenum above the source room, it decays strongly with increasing distance from the common wall. The decay is larger for the 15 cm mineral wool layer. It can also be noticed that the cavity absorption leads to a more diffuse sound field in the receiving room.

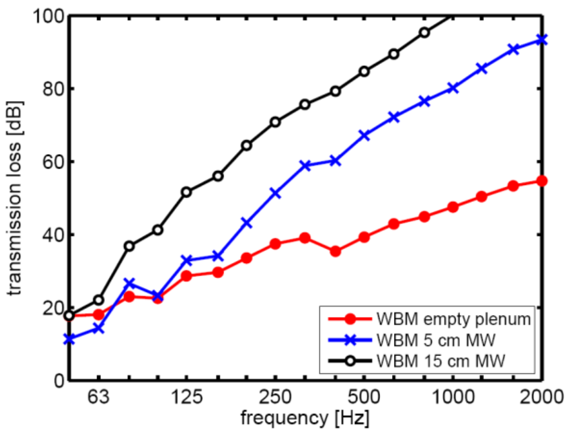


Figure 4 –TL of the airborne flanking path across a suspended ceiling with empty plenum, 5 cm or 15 cm cavity absorption modeled as equivalent fluid (Delany-Bazley model, σ = 5000 Ns/m^4).

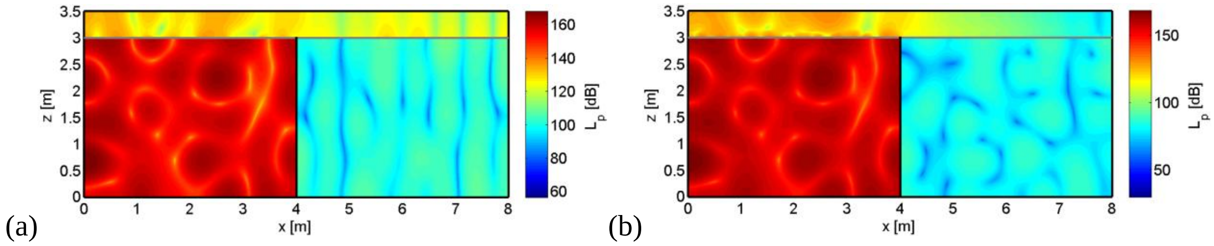


Figure 5 – Sound pressure level distribution in the plane $y = 1$ m at a frequency of 300 Hz for the suspended ceiling with (a) 5 cm cavity absorption and (b) 15 cm cavity absorption.

3.3 Influence of plenum height

Figure 6a shows the influence of the plenum height on the airborne flanking transmission when the plenum is empty. The case with plenum height 0.5 m discussed in section 3.1 is taken as the reference. The plenum height is changed to 0.3 m, 0.65 m and 0.8 m. The reverberation time of the empty plenums is changed to 0.6 s, 1.3 s and 1.6 s, respectively, so that the equivalent absorption area is the same. According to Eq. (22), the TL does not depend on the plenum height as long as the plenum absorption is the same. The results in figure 4 show however that the plenum height can significantly influence the sound insulation. Up till 300 Hz, the lower the plenum, the lower the TL. At higher frequencies, the variation with plenum height does not show any general trend. The lowest TL is obtained with a plenum height of 50 cm in this case. Decreasing the plenum height to 30 cm increases the TL at 400 and 500 Hz due to the increase in cut-on frequency. However, no clear dip in TL is visible around the cut-on frequency for the cases h = 65 cm and h = 80 cm. The plenum height of 65 cm results in the highest TL.

Figure 6b shows the variation of TL with plenum height when the plenum is filled for 30% with mineral wool. In this case, the sound insulation increases with plenum height in the entire frequency range of interest. This is mainly due to the increased thickness of the absorbing layer. When the thickness of the mineral wool is kept constant (for which the results are not shown for brevity), the TL increases with decreasing plenum height. The TL is thus not determined by the plenum height, but depends mainly on the filling ratio and the absolute thickness of the absorbent layer. Hamme [2] indicates that the influence of plenum height is much lower when the plenum is of infinite extent or has sidewall linings.

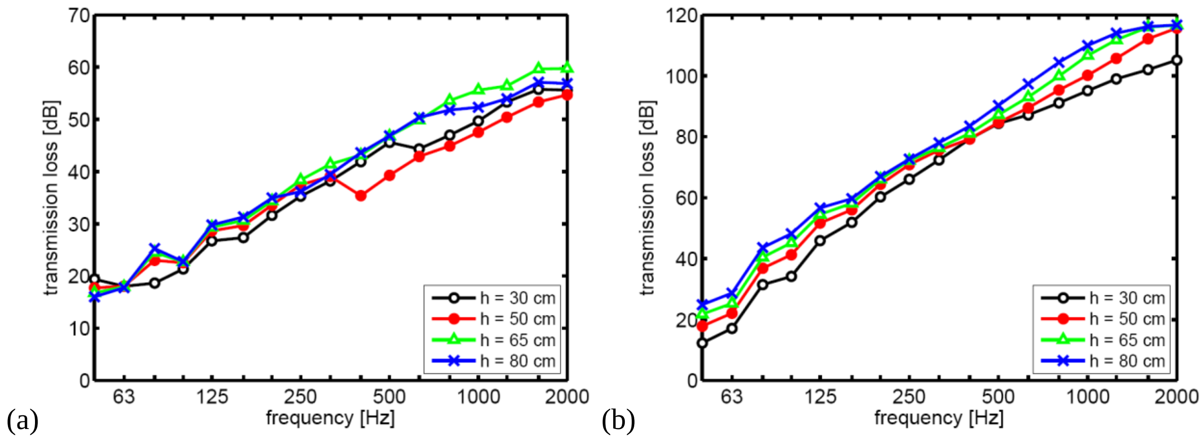


Figure 6 –Influence of the plenum height on the TL of the airborne flanking path across a suspended ceiling with (a) empty plenum and (b) mineral wool in the plenum (filling ratio 30 %).

4 Conclusions

In this paper, the wave based methodology is extended for the investigation of the airborne flanking transmission across suspended ceilings. An advantage of the model is that only the airborne flanking path is accounted for, which allows a theoretical investigation of geometrical and material parameters or cavity absorption. In practice, installation details like the partition ceiling junction or leakages and direct transmission also influence the sound transmission, which makes it difficult to interpret measurement results. A comparison between WBM and SEA results shows that statistical models fail to predict the sound transmission with good accuracy in a broad frequency range. The modal behaviour of the plenum is important up to higher frequencies because of the relatively small dimensions. As a result, the influence of plenum height on the TL is case specific and difficult to predict. The sound insulation can be significantly improved by adding an absorbent layer in the plenum. The TL increases with increasing thickness of the absorbent layer and with increasing filling ratio.

Acknowledgements

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