Acústica 2020
XI Congreso Ibérico de Acústica
51º Congreso Español de Acústica
TECNIACÚSTICA 2020

SECONDARY PATH INVERSION IN FILMS BASED ADAPTIVE NOISE CONTROL SYSTEMS

Branislav Vuksanovic[1], Dragana Nikolic[2]

[1]University of Portsmouth, Portsmouth, UK
{branislav.vuksanovic@port.ac.uk}
[2]Ascend Technologies Ltd, Southampton, UK
{d.nikolic@ascendtechnologies.co.uk}

Abstract
Filtered-x least mean squares (FXLMS) algorithm is widely used in many active noise control (ANC) DSP systems and applications. Although it has some shortcomings, its advantages are simplicity, robustness and relatively low computational load. A simple alteration of this technique, filtered-inverse least mean squares (FILMS) algorithm has been recently proposed to address some weak aspects of FXLMS technique. Here, an acoustic channel equaliser is designed off-line, before the start of cancellation process, and inserted in front of the secondary source with the aim to compensate for distortions of the cancelling signal introduced by the secondary path. The adaptive filter itself can now be significantly shortened compared to the one used in FXLMS as its main task is to track small changes of the plant and variations in the signal statistics, i.e. to fine tune the gain and phase of the acoustic noise cancellation system. This paper discusses some issues encountered by FILMS algorithm when the cancellation system contains non-minimum phase secondary path. Solution to unstable inverse of non-minimum phase secondary paths is suggested and some results provided in this work to illustrate the effectiveness of the proposed approach.

Keywords: active noise control, adaptive filtering.

PACS no. 43.50.ki, 07.50.Qx, 43.60.Mn

1 Introduction

Active noise control (ANC) system relies on the measurement of the initial noise and generation of artificially created anti-noise sound field that, when superimposed to the primary field, results in a combined noise field of lower intensity than the primary one. Most of the ANC systems use adaptive filters to generate anti-noise sound field. Those filters automatically search for optimum solutions, and keep track of environmental changes during system operation.
Various algorithms to update filter coefficients during the noise-cancellation process can be used in adaptive filters. The Filtered-x Least-Mean-Square (FXLMS) algorithm originally introduced by Burgess [1] and Widrow, et al. [2] is most often employed control algorithm in ANC systems. The widespread use of the FXLMS algorithm is mainly due to its simplicity, robustness and the low computational load. Although the FXLMS algorithm is computationally simple, its convergence speed can be slow and signal dependent. This has motivated researchers to look for algorithms with improved

convergence rates. If FIR filter structure is used, the convergence rate can be improved by using variable step-size Least Mean Square (LMS) [3], [4], Newton [5], Kalman, or Recursive Least Square (RLS) algorithms [6]. The other approach is to condition the reference signal by employing different filter structures such as lattice filter, subband filter, or orthogonal transform. Different approaches have resulted in a number of ANC algorithms with improved convergence properties: lattice ANCS, frequency-domain ANCS, RLS-based algorithms called Filtered-x RLS (FXRLS), Filtered-x Fast-Transversal-Filter (FXFTF), and IIR-filter-based LMS algorithms known as Filtered-u Recursive LMS (FURLMS), and Filtered-v algorithms.

2 Filtered LMS Algorithm (FILMS)

Filtered-inverse least means squares (FILMS) algorithm proposes the use of equalisation filter $H(z)$ whose frequency response is the inverse of the secondary path response $S(z)$, added to the FXLMS scheme as shown in Figure 1.

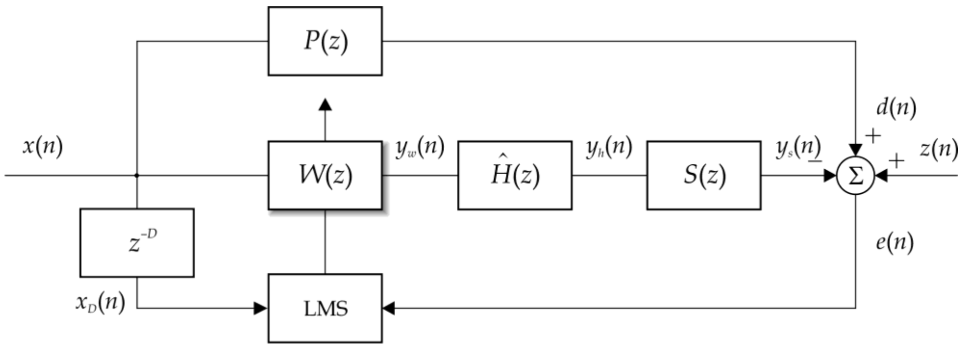


Figure 1. Block diagram of the FILMS adaptive algorithm

This acoustic channel equaliser can be designed off-line, before the start of cancellation process, and inserted in front of the secondary source with the aim to compensate for distortions of the cancelling signal introduced by the plant. This should suffice for many situations where the secondary plant is relatively constant and varies only slowly. The length of the adaptive filter used in the original FXLMS set-up can now be significantly reduced. The main task of this short adaptive filter is to track small changes of the plant and variations in the signal statistics, i.e. to fine tune the gain and phase of the cancellation system. Thus, all that needs to be done, once the secondary path influence is compensated for, is to negate and align the primary and secondary source signals to achieve the noise cancellation at the error sensor. The secondary audio plant usually consists of the audio amplifier, speaker crossover network, speaker and the acoustic transfer function between the speaker and the error microphone in the room.

Another improvement achieved with the FILMS algorithm is due to the fact that the combination of the estimates of the non-causal inverse filter and the secondary plant will have flat, uniform frequency response, thus it will eliminate the large eigenvalue spread caused by the model of the secondary plant. The convergence speed of the FXLMS algorithm is mostly dependent on the eigenvalue spread of the autocorrelation matrix of the reference signal filtered through the model of the secondary plant. This eigenvalue spread is caused by the statistics of the reference signal and by the frequency response of the secondary path model used to filter the reference signal in the FXLMS-based algorithms. A non-uniform spectrum of the reference signal as well as a non-uniform frequency response of the secondary plant model can increase the eigenvalue spread of the autocorrelation matrix of the filtered reference signals. Using the proposed modification with the non-causal inverse filter model, the second cause of the

eigenvalue spread can be eliminated. The conditioning number of the autocorrelation matrix will therefore remain dependent only on the statistic of the reference signal itself.
The combination of the inverse plant model and the real plant in the FILMS algorithm is approximately a pure delay (approximately, because the models are never perfect). The same delay needs to be used in the update branch of the FILMS algorithm to account for the possible detrimental effects of the inverse filter at the output of adaptive filter on algorithm stability. Delaying reference signal through the update branch, rather than filtering it with the estimate of the secondary path as is done in the FXLMS scheme, provides a significant computational saving especially for the multichannel FILMS configurations. The mathematical derivation of the FILMS update equation is provided in the second part of this section.

2.1 FILMS Update Equation

The following notation is used in this section:

L_p – the length of primary path impulse response,
L_s – the length of secondary path impulse response,
L_w – the length of adaptive filter,
L_h – the length of estimated secondary path inverse,
$\mathbf{p} = \left[p_0,\ p_1,\ \ldots,\ p_{L_p-1} \right]^T$ – the primary path impulse response,
$\mathbf{s} = \left[s_0,\ s_1,\ \ldots,\ s_{L_s-1} \right]^T$ – the secondary path impulse response,
$\hat{\mathbf{h}} = \left[\hat{h}_0,\ \hat{h}_1,\ \ldots,\ \hat{h}_{L_h-1} \right]^T$ – the estimated secondary path inverse,
$\mathbf{h} = \left[h_0,\ h_1,\ \ldots,\ h_{L_h-1} \right]^T$ – the exact secondary path inverse,
$\mathbf{w}(n) = \left[w_0(n),\ w_1(n),\ \ldots,\ w_{L_w-1}(n) \right]^T$ – the adaptive filter coefficients vector,
$\mathbf{x}(n) = \left[x(n),\ x(n-1),\ \ldots,\ x(n-L_p+1) \right]^T$ – reference signal vector,
$\mathbf{x}_{\hat{D}}(n) = \left[x_{\hat{D}}(n),\ x_{\hat{D}}(n-1),\ \ldots,\ x_{\hat{D}}(n-L_w+1) \right]^T$ – delayed reference signal vector,
$x(n)$ – reference signal,
$x_{\hat{D}}(n)$ – $\widehat{D}$ samples delayed reference signal,
$d(n)$ – desired signal,
$e(n)$ – error signal,
$y_w(n)$ – adaptive filter output signal,
$y_h(n)$ – inverse filter output signal,
$y_s(n)$ – secondary path output signal,
$z(n)$ – measurement noise signal.

All systems are assumed to be linear time-invariant with transfer functions modelled by linear FIR filters. The lengths of the primary path impulse response **p** and the adaptive filter coefficients vector **w**(n) are assumed to be identical, i.e. L_p=L_w. However, different filter lengths can be modelled by adding zero-valued coefficients to the impulse responses. The reference signal $x(n)$ is assumed to be the wide-sense stationary zero-mean white noise signal with variance σ_x^2. The measurement noise $z(n)$ is also taken to be the zero-mean white noise signal with variance σ_z^2 independent of other signals in the adaptation process.
Setting up the inverse model of the secondary path to be an L_h-th order FIR filter $\hat{\mathbf{h}}$, the estimation error $e(n)$, according to the scheme presented in Figure 1, can be written as:

$$\begin{aligned} e(n) &= d(n) - \sum_{i=0}^{L_s-1} s_i y_h(n-i) + z(n) \\ &= d(n) - \sum_{i=0}^{L_s-1}\sum_{j=0}^{L_h-1} s_i \hat{h}_j y_w(n-i-j) + z(n) \\ &= d(n) - \sum_{i=0}^{L_s-1}\sum_{j=0}^{L_h-1} s_i \hat{h}_j \mathbf{x}^{\mathbf{T}}(n-i-j)\mathbf{w}(n-i-j) + z(n) \end{aligned} \quad . \qquad (1)$$

The gradient vector of the estimation error can then be derived as:

$$\nabla e(n) = \begin{bmatrix} -\sum_{i=0}^{L_s-1}\sum_{j=0}^{L_h-1} s_i \hat{h}_j x(n-i-j) \\ -\sum_{i=0}^{L_s-1}\sum_{j=0}^{L_h-1} s_i \hat{h}_j x(n-i-j-1) \\ \vdots \\ -\sum_{i=0}^{L_s-1}\sum_{j=0}^{L_h-1} s_i \hat{h}_j x(n-i-j-L_w+1) \end{bmatrix} = -(\mathbf{s} * \hat{\mathbf{h}}) * \mathbf{x}(n) \quad . \qquad (2)$$

A signal $x_{sh}(n)$ depicts a result of filtering the reference signal $x(n)$ through the cascade of filters **s** and $\hat{\mathbf{h}}$, given by:

$$x_{sh}(n) = (\mathbf{s} * \hat{\mathbf{h}}) * x(n) = \sum_{i=0}^{L_s-1}\sum_{j=0}^{L_h-1} s_i \hat{h}_j x(n-i-j) \qquad (3)$$

or, in the vector form:

$$\mathbf{x}_{sh}(n) = (\mathbf{s} * \hat{\mathbf{h}}) * \mathbf{x}(n) \quad . \qquad (4)$$

If the filter $\hat{\mathbf{h}}$ is an exact inverse of the secondary path impulse response **s**, $\hat{\mathbf{h}} \equiv \mathbf{h}$, the result of convolution between those two filters is an ideal Dirac pulse, $\delta(n–D)$, where D represents an overall delay through the cascade of **s** and **h**, so $A = x_{sh} = x(n-D)$, or in the vector form $\mathbf{x}_{sh} = \mathbf{x}(n-D)$.
Similarly to the FXLMS algorithm, the coefficients of the FIR filter are selected to minimize the power of the error signal, i.e. the MSE, according to the given criterion defined by a cost function $\xi(n)$. The simple choice of $\xi(n)$ is to use the instantaneous squared error $e^2(n)$, such that the gradient vector of a cost function can be computed as:

$$\nabla\hat{\xi}(n) \equiv \nabla\left[e^2(n)\right] = 2e(n)\nabla e(n) \quad . \qquad (5)$$

Introducing a vector of the delayed reference signal $\mathbf{x}_D(n)$ such that:

$$\mathbf{x}_{sh}(n) = (\mathbf{s} * \mathbf{h}) * \mathbf{x}(n) = \delta(n-D) * \mathbf{x}(n) = \mathbf{x}(n-D) \equiv \mathbf{x}_D(n) \qquad (6)$$

the gradient vector can be written as:

$$\nabla\hat{\xi}(n) \equiv \nabla\left[e^2(n)\right] = 2e(n)\nabla e(n) = -2e(n)\mathbf{x}_D(n) \qquad (7)$$

Adjusting the coefficients of adaptive filter $\mathbf{w}(n)$ in the opposite direction of the gradient vector $\nabla\xi(n)$, i.e. using a steepest decent rule, a recursive relation for the update of the filter weights is obtained:

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{\mu}{2}\nabla\xi(n) = \mathbf{w}(n) + \mu e(n)\mathbf{x}_D(n) \tag{8}$$

The algorithm derived by this modification of the FXLMS adaptive scheme is referred to as Filtered-Inverse LMS (FILMS) algorithm.

3 Inverting the non-minimum phase secondary paths

The phase response of the plant has important effects on the design of inverse filter required by the FILMS algorithm. If the plant zeros are contained within the unit circle, the plant is of minimum phase type, i.e. the corresponding inverse filter's poles are contained within the unit circle and the inverse is stable. For a given plant's magnitude response, a minimum-phase plant is a causal system with the smallest phase response possible. A non-minimum phase plant however, has one or more zeros outside the unit circle. The inverse filter for a non-minimum phase plant has poles outside the unit circle and can therefore be unstable. This clearly represents the problem with FILMS algorithm.
This section illustrates the behaviour of the FILMS algorithm when the system with non-minimum phase secondary path is encountered. The non-minimum phase filter contains zeros outside the unit circle which would, after inversion, become unstable poles and would therefore severely degrade performance of the FILMS-based ANC system. To overcome the instability problem caused by the plant zeros outside the unit circle, non-minimum phase plant is decomposed into cascade of two filters – minimum phase (MP) and all-pass filter [7]. Minimum phase filter obtained through this decomposition is then inverted resulting in the stable inverse. The pre-filtering of the reference signal should in this situation be done through the cascade of the plant estimate and the inverted minimum-phase filter. This approach is indicated in Figure 2.

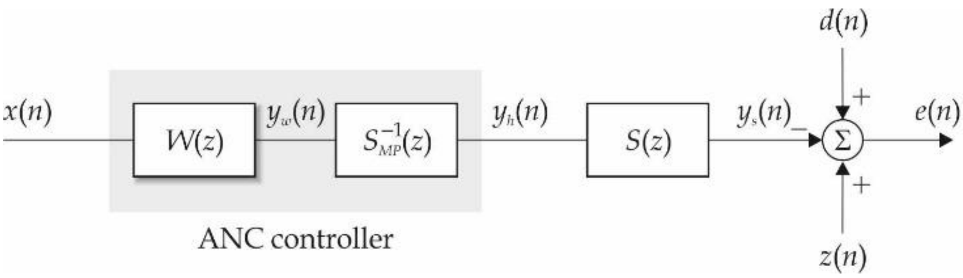
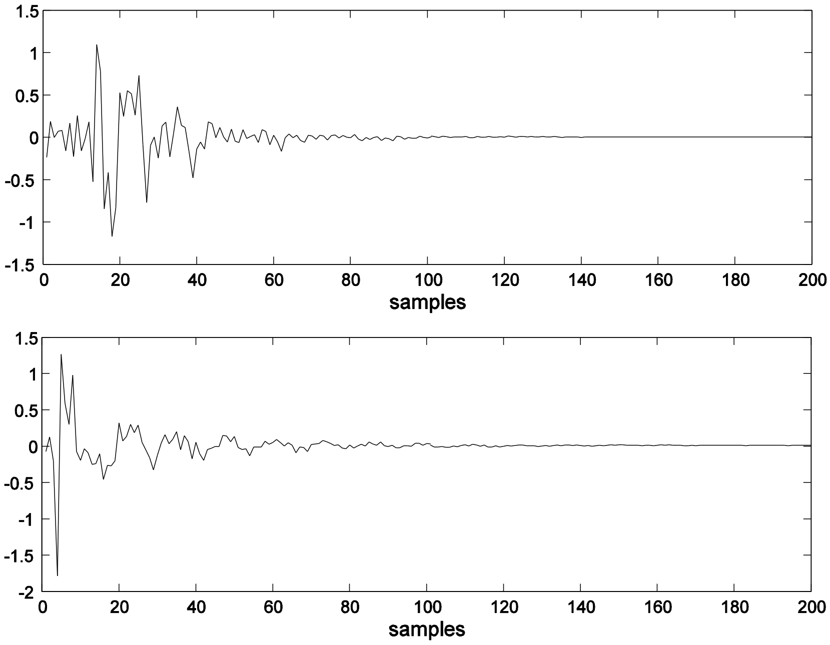


Figure 2. Modified FILMS structure

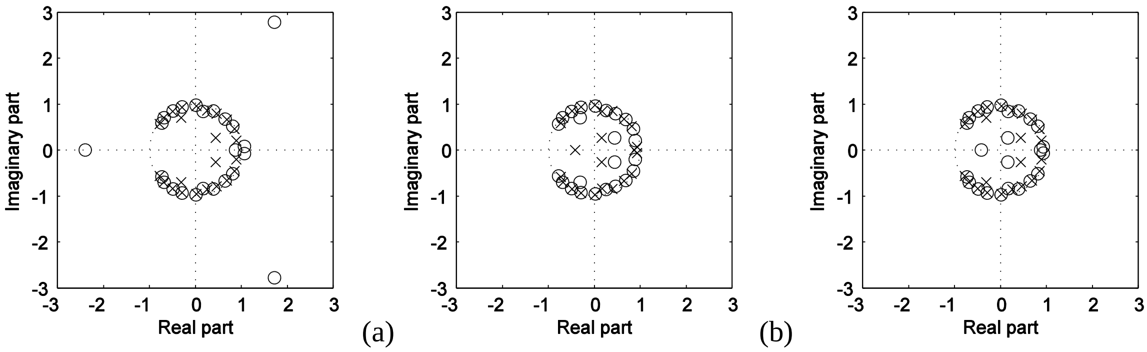
Adaptive filter will in that case converge to the realizable inverse of the all-pass part of the secondary path. The inverted minimum-phase part of the secondary path can be convolved with the estimate of the secondary path, thus some computational saving can still be achieved in these situations.
To illustrate how the FILMS algorithm performs in this special case and evaluate its performance, the secondary path with an impulse response containing a number of zeroes outside the unit circle, available in [8], has been used in this example. The employed impulse responses of the primary and secondary paths are shown in Figure 3 while the zero-pole diagrams for both secondary path and its minimum-phase inverse counterpart are given in Figure 4



(a)

(b)

Figure 3. Impulse responses of the (a) primary and (b) secondary path



(a) (b) (c)

Figure 4. Zero-pole diagram of (a) the secondary path, (b) the inverse of its minimum-phase part and (c) its minimum-phase part

Magnitude and phase characteristics of the secondary path and the inverses of the MP and AP filters are shown in Figure 5.

Performance of both the FXLMS and FILMS based active noise control systems are evaluated on this system. The step sizes used here were set to be the maximum step size for which each individual algorithm converges. The length of adaptive filter was set to 200 taps. Learning curves for both algorithms are shown in Figure 6 indicating faster convergence of the FILMS-based system. An approximately 30 dB lower level of residual noise (MSE) at the error microphone compared to the error achieved with FXLMS-based system is achieved in this case indicating the viability of proposed approach.

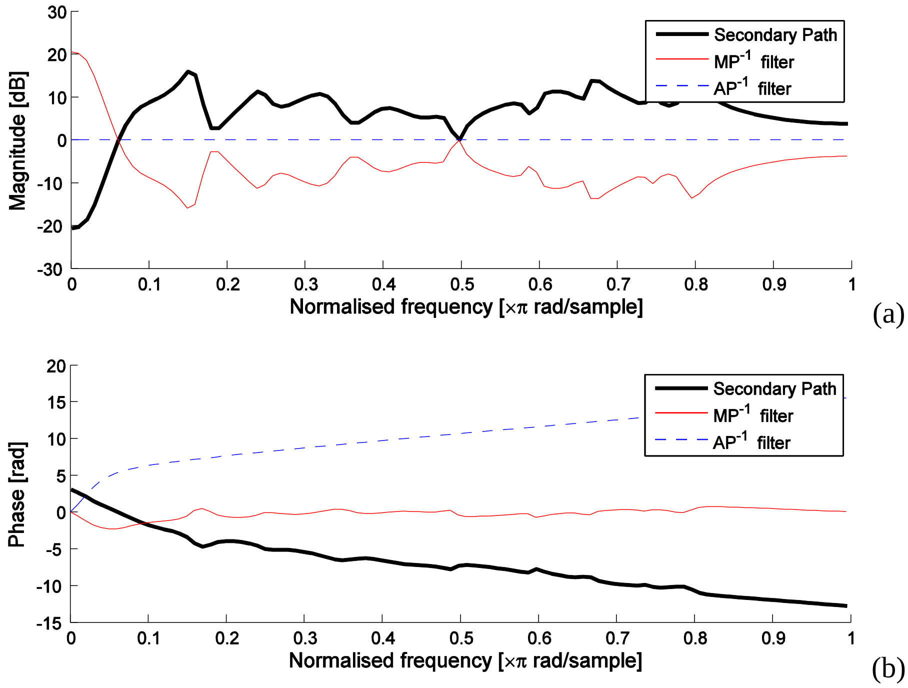


Figure 5. (a) Magnitude and (b) phase of the impulse responses of the secondary path and the inverses of the MP and AP filters

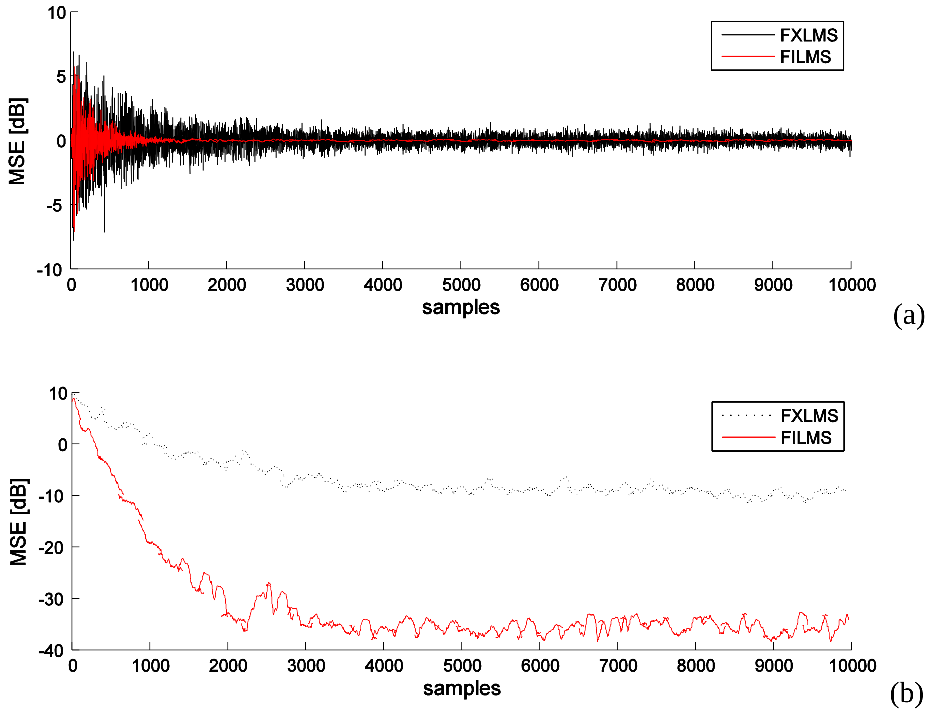


Figure 6. Comparison of the FXLMS and FILMS algorithm performances: (a) the error signal and (b) MSE

4 Conclusions

A modification of a well-known Filtered-x LMS active noise control adaptive scheme named Filtered-inverse LMS algorithm has been presented. This technique can in some situations significantly improve the rate of convergence of the FXLMS algorithm, where convergence rate remains highly dependent upon the conditioning of the autocorrelation matrix of the reference signals. FILMS algorithm has been published and discussed in some earlier papers so the focus of this work is the problem of inversion of non-minimum phase secondary path present in many active noise control applications. The non-minimum phase system contains zeros outside the unit circle, which would, after inversion, become unstable poles and would therefore severely degrade performance of the FILMS-based ANC system. To account for this behaviour, secondary path is first decomposed into minimum-phase (MP) and all-pass (AP) parts. The inverse filter used in the FILMS algorithm is than replaced with an inverse of the minimum phase (MP) part of the secondary path. The reference signal is now pre-filtered through the combination of estimate of secondary path and an inverse of the MP filter before being used to update the adaptive filter coefficients. This approach results in stable control system as well as high convergence rate. This is illustrated with the results in the last section of the paper.

Acknowledgements

This work is supported by the Royal Society International Exchanges Award IES\R2\181172.

References

[1] J. C. Burgess, "Active adaptive sound control in a duct: a computer simulation," Journal of Acoustic Society of America, vol. 70, pp. 715–726, 1981.

[2] B. Widrow, D. Schur, and S. Sohaffer, "On adaptive inverse control," presented at Proceedings of the 15th Asilomar Conference on Circuits, Systems and Computers, Pacific Grove, CA, 1981.

[3] R. H. Kwong and E. W. Johnston, "A variable step size LMS algorithm," IEEE Transactions on Signal Processing, vol. 40, pp. 1633-1642, 1992.

[4] C. G. Lopes and J. C. M. Bermudez, "Evaluation and design of variable step size adaptive algorithms," presented at Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP-2001), Salt Lake City, Utah, U.S.A., 2001.

[5] B. Widrow and S. D. Stearns, Adaptive signal processing. NJ, USA: Prentice-Hall, Englewood Cliffs, 1985.

[6] S. Haykin, Adaptive filter theory, 2nd ed. NJ: Prentice-Hall, Englewood Cliffs, 1991.

[7] A. V. Oppenheim and R. W. Schafer, Discrete-time signal processing: Prentice Hall, 2010.

[8] S. M. Kuo and D. R. Morgan, Active noise control systems: algorithms and DSP implementations. New York: John Wiley&Sons, 1996.