Direct usage of FE surface meshes and result data for Boundary Element Method

S. Schneider, S. Marburg, H.–J. Hardtke
{sneider,marburg,hardtke}@mfm.mw.tu-dresden.de
Institut für Festkörpermechanik
Technische Universität
01062 Dresden
Germany

Abstract

In the context of growing importance of environmental protection the calculation of radiated or scattered sound fields of technical housings is more and more important. Especially for exterior problems the Boundary Element Method (BEM) is an efficient tool for performing such a calculation. The main disadvantage of this method is it's high memory consumption because of the appearing dens (non-symmetric) system matrix. This restricts the method to problems of moderate size ($N < 10.000$). In general there exists already finite element surface meshes of the surface under consideration which consists of far more then 10.000 of unknowns. With the usage of the so called Fast Methods we are able to use directly the surface meshes and result data on it available due to a finite element structural analysis. Further we are using an ILU-type preconditioner for accelerating the iterative solution of the linear system.

1 Introduction

In the context of growing importance of environmental protection the calculation of radiated or scattered sound fields of technical housings is more and more important. With other words engineers are interested in calculating the radiated sound fields of vibrating structures already in a early design phase to build low radiating technical objects. Especially for exterior problems the Boundary Element Method (BEM) is an efficient tool for performing such a calculation. This is because of the facts that the solutions obtained by this method do satisfy a priori the Sommerfeld radiation condition and the reduction of the dimension of the problem. The main disadvantage of BEM is it's high memory consumption because of the appearing dens (non-symmetric) system matrix. This restricts the method to problems of moderate size ($N < 10.000$). In general there exists already a finite element surface meshes of the surface which consists of far more then 10.000 of unknowns. This is the result of modeling even the finest details on the surface. For an acoustic analysis of a surface these details are not important as long as their size is well bellow the wavelength (l$< \lambda/10$)

of the acoustic waves. To overcome this problem often a second much coarser fluid-mesh is build. Within this process details which are assumed to be not important are removed. The later process has to be done more or less by hand. With the usage of the so called Fast Methods we are able to use directly the FE-meshes. The coarsening process described above is realized implicitly in the fast method. The main idea of these methods is to approximate the dense matrix by sparse matrices. Hence, the original matrix $(I + A)$ is approximate by

$$I + A = I + A_{\text{near}} + A_{\text{far}} \approx I + A_{\text{near}} + CDT. \tag{1}$$

Namely, the Regular Grid Method (RGM) [1] and the Multilevel Fast Multipole Method (MLFMA) [2, 3, 4, 5, 6] are used. The memory requirement of these methods is $\sim \mathcal{O}(N)$. This enables us to solve problems with about 100.000 unknowns in core on a computer with 1 Gb main memory.

2 Iterative solution and preconditioning

When implementing the fast methods mentioned above it turns out that the iterative solution of the linear system is the most time consuming part of the whole calculation due to the slow convergence of the iterative solver[1]. One reason for the poor performance of GMRes it the fact that the matrix A in (1) is the discretization of a non-compact operator. Therefor it has an eigenvalue distribution for which iterative solver like GMRes are not suitable (see [7]). Furthermore the

	ilu1			ilu50			no		
size	setup	solve	total	setup	solve	total	setup	solve	total
3.4λ	35	22	**57**	38	9	**47**	35	115	**150**
2.5λ	30	16	**46**	32	6	**38**	30	90	**120**
2.0λ	30	11	**41**	32	4	**36**	30	75	**105**
1.2λ	30	10	**40**	32	2	**34**	30	70	**100**
$.6\lambda$	30	11	**41**	32	3	**35**	30	70	**100**

Table 1: Timing of different parts of the solution with and without preconditioning

convergence rate is influenced by the smoothness of the surface as well. To accelerate the iterative solver we apply an ILU-factorisation type preconditioner of the matrix $I + A_{\text{near}}$. The time and memory consumption for its construction and application is negligible in comparison to other parts of the calculation. Tab. 1 shows the improvings

[1] Different iterative solvers like GMRes , CGNR, TFQMR and BICGstab were investigated. GMRes turned out to be the most stable and efficient one.

of the preconditioner for the problem discussed in the following section. The slidly higher setup time is compensated by a significant reduction of the time needed for the iterative solution. Even with a very cheap preconditioner (ilu1- one element per row in the factors L and U) the total solution time was reduced by the factor three. Hence instead of solving

$$(I + A)x = b \tag{2}$$

directly we are solving the preconditioned system

$$(I + A)M^{-1}y = b \tag{3}$$

and

$$x = M^{-1}y\,. \tag{4}$$

We define the residual at the n-th iteration step as

$$\frac{||(I + A)M^{-1}y_n - b||}{||b||} = \varepsilon_n \;. \tag{5}$$

The iteration process is terminated if $\varepsilon_n < 10^{-6}$ is satisfied.

3 Numerical examples

We will calculate the radiated sound field of a three cylinder piston compressor of KNORR–Brake Munich. For a detailed description of the compressor, we refer to [8].

With the given finite element discretization we calculate the eigenfrequencies in the range of 100 to 2000 Hertz. Out of these we select eigenfrequencies with characteristic mode shapes, for example, first, second and fourth plate bending modes of the plane parts of the housing. The normal displacements of the nodes are used as normal velocities for the acoustic simulation. These values are adjusted to the values measured at the Institut für Festkörpermechanik with a laser scanning vibrometer. We measured a normal surface velocity of approximately 7 mm/s for the eigenfrequency at 1431 Hz and approximately 1 mm/s for the eigenfrequencies at 360 to 1115 and 1940 Hz. According to the experimental data, these are the dominant surface velocities.

The compressor is located one meter above the ground (the axis of the cylinders are parallel to the ground) which is assumed to be rigid.

This corresponds to the measurements performed at KNORR–Brake Munich to evaluate the radiated sound pressure level of such a compressor.
The mesh of the housing of the compressor is shown in Fig. 1. This mesh consists of 18503 triangular and rectangular elements. Element size is limited to a maximum of 2.8 cm. This guarantees at least six elements per wavelength up to a frequency of 2000 Hz. Using constant

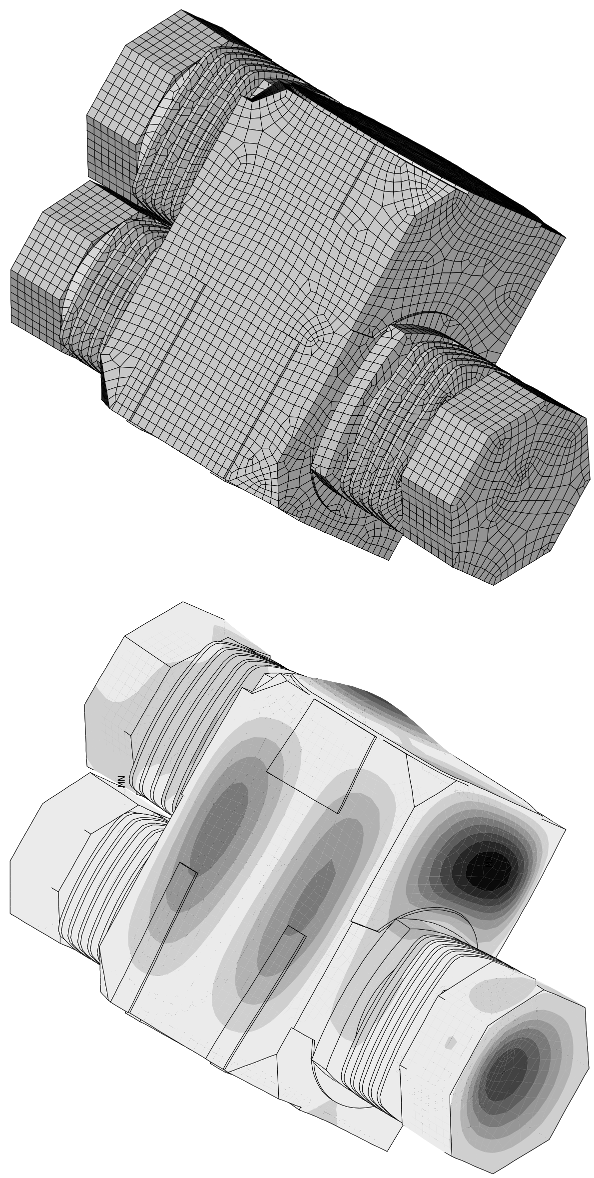

Figure 1: Finite element mesh of structure and boundary element mesh of fluid coincide (left). Mode shape for eigenfrequency at 1431 Hz (right).

acoustic boundary elements we have the same number of unknowns as surface elements for the sound pressure at the mid points of the elements. The amount of memory in use at the highest frequency is about 200 Mb. Tab. 1 shows the timing for the solution process. The columns labeled *setup* represents the time needed up to the begin of the first iteration step whereas the columns labeled *solve* give the time the iterative solver needed to reach the required residual. In the unpreconditioned case the total (setup+solve) time is dominated by the time to solve the linear system. When using the preconditioner espe-

Pre-cond.	Number of iterations				
	3.4λ	2.5λ	2.0λ	1.2λ	$.6\lambda$
no	705	600	534	509	515
ilu1	170	125	100	90	92
ilu50	77	48	33	25	23

Table 2: Number of iterations needed with and without preconditioning

cially at low frequencies the time for solving the system becomes less important. This is caused by the significant reduction of the number of iterations needed (see Tab. 2).

4 Conclusion

The application of methods like Regular Grid Method or Multilevel Fast Multipole Method enables us to overcome the high memory consumption of the standard Boundary Element Method. In consequence much larger problems can be solved in core. The most time consuming part of the BE-calculation , -the iterative solution of the linear system-, was successfully shortened by the application of a preconditioner.
When combining the last two techniques we are able to use FE-surface meshes and nodal data on it directly for an acoustic analysis using the Boundary Element Method.

Acknowledgement

The authors wish to thank KNORR–Brake Munich for providing the CAD model of the piston compressor. Furthermore, it is acknowledged that the computation was run on the SGI Origin 2000 at the Zentrum für Hochleistungsrechnen of the Technische Universität Dresden.

References

[1] A. Bespalov. On the usage of a regular grid for implementation of boundary integral methods for wave problems. *Russ. J. Numer.*

Anal. Math. Modelling, 15(6):469–488, 2000.

[2] V. Rokhlin. Diagonal Forms of the Translation Operators for the Helmholtz Equation in Three Dimensions. *Applied and Computational Harmonic Analysis*, 1:82–93, 1993.

[3] M. F. Gyure and M. A. Stalzer. A prescription for the Multilevel Helmholtz FMM. *IEEE Computanional Science & Engineering*, 5(3):39–47, 1998.

[4] K. Giebermann. *Schnelle Summationsverfahren zur numerischen Lösung von Integralgleichungen für Streuprobleme im* $\mathbb{R}^3$. PhD thesis, Universität Karlsruhe, 1997.

[5] W. C. Chew, J. M. Jin, C. C. Lu, E. Michelssen, and J. M. Song. Fast Solution Methods in Electromagnetics. *IEE Trans. Antennas Propag.*, 45(3):533–543, 1997.

[6] J. Rahola. Diagonal Forms of the Translation Operators in the Fast Multipole Algorithm for Scattering Problems. *BIT*, 60(2):333–358, 1996.

[7] K. Burrage and J. Erhel. On the performance of various adaptive preconditioned GMRES strategies. *Numerical linear algebra with applications*, 5(2):101–121, /1998.

[8] F. Meyer M. Hartl and S. Schneider. Oil-free low-vibration piston compressor in railway applications. In *Conference on compressors and their systems*, pages 177–188. IMechE Conference Transactions, Professional Engineering Publishing, 2001.