A REVIEW OF GROUND IMPEDANCE MODELS FOR PROPAGATION MODELLING

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ABSTRACT

The acoustical properties of the ground surface are considered in the light of models for the acoustical properties of rigid porous media and recent data. The influence of small-scale roughness provides an alternative mechanism to layering or decreasing porosity with depth by which the real part of ground impedance is less than the imaginary part. A full description of the acoustical properties of an outdoor ground surface requires information about the mean height and spacing of surface roughness as well as porosity, tortuosity, connected pore geometry, flow resistivity and layering. Simplified models are described and conclusions about ground impedance modelling are drawn.

INTRODUCTION

The propagation of sound near the ground depends on the surface impedance. Surface porosity and associated air permeability allows sound to penetrate and hence to be both absorbed and undergo phase change through friction and thermal exchanges between the pore fluid and the surrounding solid. The usual assumption in outdoor acoustics is that the ground has a rigid-frame and that incident sound waves do not cause motion of the solid particles as well as the pore fluid. The role of ground elasticity is not pursued further in this review. There is interference between sound travelling directly between source and receiver and sound reflected from the ground. This interference is known as ground effect [1]. Over porous surfaces, similar enhancement to that expected over an acoustically-hard surface tends to occur at low frequencies since the longer wavelengths are less able it is to penetrate the pores. In general, the surface impedance of a rigid porous material with a smooth surface depends on the surface porosity (of air-filled pores connected to the surface), tortuosity (i.e. the twistiness) of pores, the flow resistivity (related to the inverse of air permeability) and near-surface layering (or porosity profile with depth). Typically, the impedance of porous surfaces decreases with increasing frequency. Tortuosity and porosity have their main influence at higher frequencies (>1 kHz). The importance of layering, or of variation in properties with depth, depends on the surface flow resistivity (related to inverse of air permeability). If the ground has very high flow resistivity, like wet compacted clay or silt, and only low frequencies are of interest, then it is possible to predict its surface impedance from its flow resistivity alone. A widely-used empirical model for ground impedance requires a single parameter, the effective flow resistivity [2], but has been found to predict an incorrect frequency-dependence at low frequencies [3]. In recent numerical modelling [4], use has been made of a three-parameter phenomenological model. If the ground may be

treated as homogeneous or semi-infinite then, strictly, only an expression for characteristic impedance (Z_c) is needed. However, if a near-surface layer influences sound reflection, then an expression for propagation constant (k) in the layer is needed together with the surface layer thickness. Near-grazing propagation of sound depends also on the surface roughness. Roughness that has characteristic dimensions less than the wavelengths of interest is known to influence the effective impedance of the surface. Surface roughness results in incoherent scattering also. This becomes increasingly important as the mean roughness size increases compared with the incident wavelengths. At ranges of several km, the high-frequency behaviour of ground impedance is of little consequence. It is likely that the ground type will change as a function of range. In some situations an 'area-average' will be the most relevant value. The standard 'template' method for deducing ground impedance relies on fitting short-range measurements of excess attenuation or level difference spectra [5]. Extrapolation to frequencies outside the range of the measurement will depend on the accuracy of the model used.
This paper reviews several impedance models and roughness-induced effective impedance and offers conclusions and recommendations for ground impedance representation in outdoor propagation modelling.

IMPEDANCE MODELS

Semi-empirical model A model for the propagation constant (k) and characteristic impedance relative to air (Z_c) of a semi-infinite rigid porous layer developed on the basis of semi-empirical considerations and involving many measurements on fibrous materials [2] has been used widely for modelling the impedance of outdoor ground surfaces. It may be written

$$k = (\omega/c_f)\left[1 + 0.0978X^{-0.700} + i0.189X^{-0.595}\right] \qquad (1a)$$

$$Z_c = 1 + 0.0571\,X^{-0.754} + i0.087\,X^{-0.732} \qquad (1b),$$

where ω is angular frequency, $X = \rho_f f/\Omega R_s$ = f/R_{eff}; ρ_f is fluid density (kg m^{-3}) (assumed to be unity), f is frequency (Hz), Ω is porosity, Rs is flow resistivity (Pa s m^{-2}) and R_{eff} is an effective flow resistivity (Pa s m^{-2}). Time dependence $e^{-i\omega t}$ is understood.

Phenomenological models
The phenomenological model for a rigid-porous medium [6] may be expressed

$$k = \sqrt{\gamma}\,\frac{\omega}{c_f}\sqrt{T + \frac{iR_s\Omega}{\omega\rho_f}} \qquad (2a),$$

$$Z_c = \frac{1}{\Omega\sqrt{\gamma}}\sqrt{T + \frac{iR_s\Omega}{\omega\rho_f}} \qquad (2b)$$

where T is tortuosity. The phenomenological model ignores the frequency dependence of the bulk modulus of air in the pores resulting from thermal exchange between air and solid. Equations can be written either assuming adiabatic or isothermal conditions for wave propagation within the porous medium. Equations (2a) and (2b) are based on the isothermal assumption. They can be deduced directly as low frequency/high flow resistivity approximations of the Biot/Stinson/Champoux microstructural model [7]. The adiabatic assumption leads to an expression for impedance that is a factor of $\sqrt{(\gamma)}$ greater than that given by (2b) and gives improved agreement with data at high frquencies. Hamet [8] has used a modified form of this model that takes account of frequency-dependent heat transfer effects to predict the acoustical properties of porous asphalt. His model may be written

$$Z_c = \left(\frac{1}{\Omega}\right)\left(\frac{T}{\gamma}\right)^{1/2}\left\{1 + \frac{\gamma - 1}{\gamma}\left(\frac{1}{F_0}\right)^{1/2}\right\}F_m^{1/2},\ k = \gamma\Omega k_0 Z_c \qquad (3a)$$

$$F_m = 1 + i\omega_m/\omega,\ F_0 = 1 + i\omega_0/\omega,\ \omega_m = (R_s/\rho_0)(\Omega/T),\ \omega_0 = \omega_m(T/N_{PR}). \qquad (3b)$$

where N_{PR} is the Prandtl number for air. The phenomenological models are three-parameter models.

Microstructural models and their approximations The acoustical properties of a rigid porous medium with porosity Ω and tortuosity T containing slit-like pores may be expressed [7]:-

$$k = \omega[T\rho(\lambda)C(\lambda)]^{0.5} \quad (4a)$$

$$Z_C = (\rho_f c_f)^{-1}[(T/\Omega^2)\rho(\lambda)/C(\lambda)]^{0.5} \quad (4b)$$

$$\rho(\lambda) = (T/\Omega)\rho_f[1 - \tanh(\lambda\sqrt{-i})/(\lambda\sqrt{-i})]^{-1} \quad (4c)$$

$$C(\omega) = (\gamma P_0)^{-1}[\gamma - (\gamma - 1)H(\lambda\sqrt{(N_{PR})})] \quad (4d)$$

$$H(\lambda) = 1 - \tanh(\lambda\sqrt{-i})/(\lambda\sqrt{-i}) \quad (4e)$$

$$\lambda = \left(\frac{3\rho_f \omega T}{\Omega R_S}\right)^{1/2} \quad (4f)$$

where $(\gamma P_0)^{-1} = (\rho_f c_f^2)^{-1}$.

Since these only use hyperbolic functions, they are eminently suitable for rapid computation without approximation. Nevertheless equations 4a and 4b may be approximated to give

$$k = \sqrt{\gamma}\frac{\omega}{c_f}\sqrt{\left[\left(\frac{6}{5} - \frac{(\gamma-1)}{\gamma}N_{PR}\right)T + i\frac{4R_{eff}}{\omega\rho_f}\right]} \quad (5a)$$

$$Z_c = \left[\frac{\frac{6}{5}T\rho_f + i\frac{4R_{eff}}{\omega}}{\Omega k}\right]\frac{\omega}{\rho_f c_f} \quad (5b).$$

where effective flow resistivity $R_{eff} = \Omega R_s/4$**.** This represents a three-parameter approximation applicable to a medium containing pores of arbitrary shape. A further approximation for high flow resistivity and low frequency, gives [9,10]

$$Z_c = \frac{1}{2\sqrt{(\pi\gamma\rho_f)}}(1+i)\sqrt{\left(\frac{R_s}{\Omega f}\right)} \quad (6).$$

This may be regarded as a single parameter model for the surface impedance of a homogeneous semi-infinite rigid-porous medium with high flow resistivity, (and/or at low frequency), where the single parameter is effective flow resistivity (R_S/Ω). It predicts surface impedances in which real, R, and imaginary, X, parts are equal and decrease as the square root of frequency.

For a medium consisting of identical pores of arbitrary shape, equation (4c) may be written:-

$$\rho(\omega) = (T\rho_0/\Omega)[1 + F(\varepsilon)/(T\varepsilon^2)] \quad (7a)$$

where $\varepsilon = \lambda\sqrt{(-i)}$ and $F(\varepsilon)$ is the viscosity correction function. This format has been used in modelling acoustical properties of a medium containing pores of identical shape but with a log-normal distribution of pore sizes [11,12]. Low- and high-frequency asymptotes for the viscosity correction function may be expressed as

$$F(\varepsilon) = 1 + \theta_1\varepsilon^2 + O(\varepsilon^4), \quad \varepsilon \to 0 \quad (7b)$$

and

$$F(\varepsilon) = \theta_2\varepsilon + O(1), \quad \varepsilon \to \infty . \quad (7c)$$

Hence, a Padé approximation for the viscosity correction function in a medium with a log-normal pore size distribution has been proposed in the form

$$F(\varepsilon) = \frac{1 + a_1\varepsilon + a_2\varepsilon^2}{1 + b_1\varepsilon} \quad (7d)$$

where $a_1 = \theta_1/\theta_2$, $a_2 = \theta_1$ and $b_1 = a_1$. Values of these coefficients have been determined analytically only for certain pore shapes. However it can be shown that, for a given flow resistivity, the influence of pore shape *per se* on impedance is relatively small. Other relevant modelling developments include the (two parameter) relaxation model [13] and a model for the acoustical properties of a medium consisting of spherical grains in terms of grain diameter and porosity [14].

Only relatively smooth outdoor ground surfaces of high flow resistivity (such that airborne sound does not penetrate beyond a depth of a centimetre or two) conform to models semi-infinite media. Many outdoor grounds have near-surface layers or porosity profiles that influence sound reflection and have surfaces with significant roughness.

Models including layers The impedance of a locally-reacting porous layer above an acoustically-hard surface is given by [9]

$$Z(d) = Z_c \coth(ikd), \qquad (8)$$

where d is the layer thickness. For layered ground, such as mature forest floors, where the flow resistivity of the surface porous (litter) layer is low, and it lies above a porous substrate with finite impedance, a multi-layer model may be more suitable. A non-hard backed layer model may be deduced, from transmission line analysis, and is expressed by means of the following equation:-

$$Z(d) = Z_1\{[Z_2 - iZ_1 \tan(kd)]/[Z_1 - iZ_2 \tan(kd)]\}, \qquad (9)$$

where Z_1 and Z_2 are the relative characteristic impedances of the upper porous layer and (semi-infinite) porous substrate, respectively, k is the propagation constant in the upper porous layer and d is the layer thickness. A two-parameter approximation for multi-layered ground or with a continuously (exponential) increasing porosity with depth [10,15] may be written generically in the form

$$Z = a(1+i)\sqrt{\frac{R_e}{f}} + \frac{ib\boldsymbol{a}_e}{f}, \qquad (10)$$

where $a = 1/\sqrt{\boldsymbol{pr}_f g}$, $b = c_f/8\boldsymbol{p}g$ and $R_e = R_S/4W$. For a non-hard backed thin layer, $\boldsymbol{a}_e = 1/d_e$ with $d_e = Wd$. An approximation for the surface impedance of a high flow resistivity porous medium with the porosity *increasing* exponentially with depth is given by (10) with negative $\boldsymbol{a}_e$. If $\boldsymbol{a}_e$ is negative then (10) predicts a resistance that exceeds the reactance at all frequencies.

Rough surface effects As remarked earlier, most outdoor ground surfaces exhibit small-scale roughness. Boss theories, borrowed from underwater acoustics, have been adapted to model the effect of surface roughness on ground impedance [16 – 18]. For sound incident at grazing angle $\boldsymbol{a}$ normal to 2D-roughness axes on a porous boundary, the effective admittance may be written

$$\boldsymbol{b}^* = \frac{k_0{}^3 bV^2}{2}(1-W)^2\left[1+\left(\frac{\boldsymbol{d}^2}{2}\right)\sin^2\boldsymbol{a}\right] - ik_0V\left[\boldsymbol{d}\sin^2\boldsymbol{a} - 1 + g\Omega\right] + \boldsymbol{b}_s \,. \qquad (11)$$

where k_0 is the wavenumber in air, b is the mean centre-to-centre roughness spacing, V is the protruding cross-sectional area of roughness per unit length of imbedding plane, W is a randomness factor (=1 for periodic and 0 for random), $\boldsymbol{d} = 2s_2/\boldsymbol{n}_2$, $s_2 = \frac{1}{2}(1+K)$, K depends on roughness shape, $\boldsymbol{n}_2 = 1 + \frac{2\boldsymbol{p}V}{3b}s_2$, and $\boldsymbol{b}_s$ is the admittance of the smooth porous surface. A useful approximation for the effective impedance of a rough porous surface is

$$Z_r \approx Z_s - \left(\frac{\langle H\rangle R_s}{\boldsymbol{gr}_0 c_0}\right)\left(\frac{2}{\boldsymbol{n}} - 1\right), \qquad (12)$$

where $\boldsymbol{u} = 1 + \frac{2}{3}\boldsymbol{p}\langle H\rangle$ and $\langle H\rangle$ is rms roughness height. The usefulness of this approximation for frequencies less than 1000 Hz is indicated in Figure 1. This figure indicates also that, below 1000 Hz it is adequate to use the isothermal (3 parameter) phenomenological model to calculate Z_s. It can be shown that such a model predicts relative magnitudes of real and imaginary parts of impedance similar to those predicted by equation (10).

COMPARISONS WITH DATA

Figure 2 compares impedance data obtained using an impulse method [19] and best-fit predictions using the semi-empirical model (equation (1)) and the slit pore model (equation (4)). The semi-empirical model predicts the wrong frequency-dependence at low frequencies. Figure

3 compares impedance deduced from complex excess attenuation data obtained over established grass and predictions of the rough surface model (equation (12)) [18,20]. The model predicts the observed tendency of the data towards zero impedance at high frequencies. Elsewhere [18,21] it has been shown that the inclusion of roughness effects in the impedance model is able to explain data obtained over ground that has been roughened by farming processes.

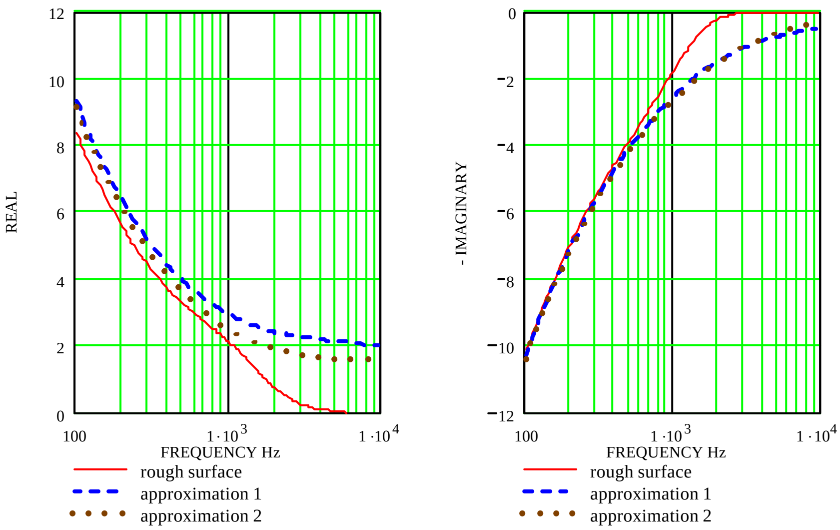

Fig. 1.- Comparison between predictions of the complex relative surface impedance of a rough surface (flow resistivity 100 kPa s m^{-2}, porosity 0.4 and tortuosity 2.5, randomly-spaced semi-cylindrical roughness radius 0.04 m, mean spacing 0.16 m) and two approximations based on equation (12). Approximation 1 uses with the 'exact' smooth surface impedance for Z_s; approximation 2 uses the isothermal phenomenological model for Z_s.

R

- X

Fig. 2.- Measured relative surface impedance of a compacted soil (squares) and predictions using the semi-empirical model (broken lines) using effective flow resistivity 450 kPa s m^{-2}. and the slit pore model (equations (3)) for a semi-infinite medium with flow resistivity 250 kPa s m^{-2}, porosity 0.5, tortuosity 2.

CONCLUSIONS

Template methods of ground impedance characterisation are now standard (e.g. [5]). However these rely on the accuracy of the impedance models on which the templates are based. The most reliable information about ground impedance is likely to be given by impedance deduction from measurement [21]. Current knowledge of the acoustical properties of rigid-framed porous surfaces suggests that a full description of ground impedance requires knowledge of porosity, flow resistivity, tortuosity, pore-size distribution, near-surface layering and surface roughness (characterised by mean height and spacing). This is unlikely to be practicable for most outdoor sound modelling. On the other hand there is considerable evidence that the semi-empirical model (equation (1)) predicts the wrong low-frequency behaviour compared with data below 1000 Hz([3] and Fig.2). At higher frequencies with adjustable parameters similar predictions to those of the semi-empirical model can be obtained by more physically justifiable models. So it is suggested that simple ground impedance models should be based on these.

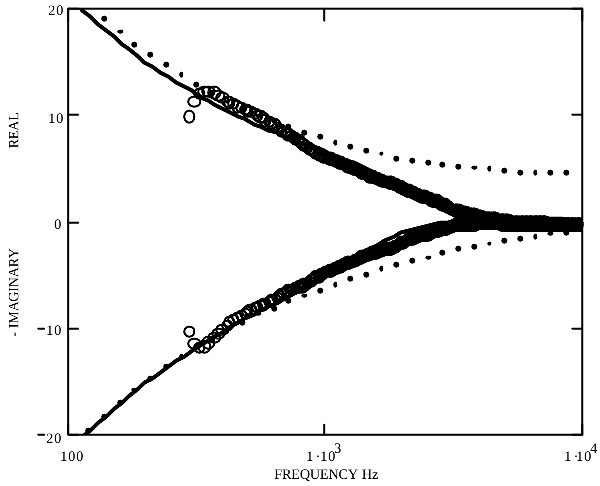

Fig. 3- Normalised impedance data (open circles) obtained from complex excess attenuation measurements over established grassland. The theoretical predictions (dotted and solid lines) are for the impedance of a semi-infinite slit pore medium, with flow resistivity 400 kPa s m^{-2}, porosity 0.4, tortuosity = 1/porosity without (dotted lines) and with (solid lines) roughness characterised by a partially-random (randomness parameter *W* = 0.5) close packed semi-cylindrical roughness with 0.02 m radius.

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