SOUND MEASUREMENT BY LASER DOPPLER ANEMOMETRY

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ABSTRACT

The use of Laser Doppler Anemometry (LDA) for sound measurement offers a number of fundamental advantages over the use of conventional microphones, although its application is also limited. One of the main advantages is that it is non-intrusive; also it does not require calibration. A major limitation, on the other hand, is that it can only be applied to periodic sound fields, indeed sound fields where there are only just a few frequency components.

This paper describes the application of LDA using the photon correlation signal analysis technique. The high sensitivity of this method means that very little or no seeding is required. The theoretical form of the Auto-Correlation Function (ACF) will be discussed. The results of experiments in both tubes and free fields will be presented and compared with conventional microphone measurements. There will also be an assessment of different signal analysis techniques such as spectrum analysis and curve-fitting.

1. INTRODUCTION

Laser Doppler Anemometry is a non-intrusive, optical technique for measuring fluid flow. The measuring volume consists of interference fringes formed by intersecting laser beams, and is shown in figure 1, together with a close-up of the interference fringes (from [1]).

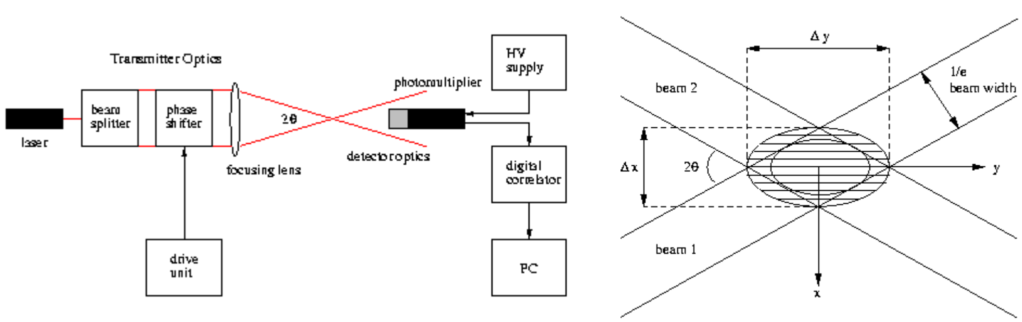

Figure 1: Photon Correlation Apparatus and Measurement Volume

The laser beam has a wavelength λ and is assumed to have a Guassian profile of $\frac{1}{e^2}$ diameter d. θ is the half-angle of the beam intersection. The measurement volume has dimensions $\Delta x = \frac{d}{\cos\theta}$, $\Delta y = \frac{d}{\sin\theta}$ and $\Delta z = d$; these values are typically fractions of a millimetre, so LDA is often termed a point measuring technique. Each of the two beams is assumed to be of equal intensity, and the spacing between consecutive fringes is $\frac{\lambda}{2\sin\theta}$.

Small particles suspended in the fluid scatter some of this light. The number of photons scattered depends on a variety of factors, such as the number and size of the particles, the intensity of the laser beam and their position within the interference pattern, and this number changes as the particles follow the fluid motion. A photomultiplier records some of this scattered light; if its intensity is sufficiently small, corresponding to an average of less than 40 million photons per second (one per 25ns), individual photons can be counted. The temporal photon distribution can then be auto-correlated, and the form of the auto-correlation function (ACF) used to determine features of the flow, such as its velocity.

The sensitivity of this technique is such that natural impurities in the fluid are frequently sufficient to produce an adequate signal, and the addition of extra seeding particles is not only unnecessary but can saturate the photomultiplier output.

2. BACKGROUND THEORY

A Digital Correlator board produces the ACF, from which various flow features can be extracted if the theoretical form of the function is known.

The scattering particles are assumed to follow the flow faithfully, and have an instantaneous velocity of the form

$$v(x) = u_0 + u_m \sin(\omega_m x) \tag{1}$$

where u_0 is the mean flow velocity, u_m is the acoustic velocity amplitude and ω_m is the acoustic angular frequency. That is, the fluid motion is considered to be a superpostion of a mean flow and an acoustic oscillation.

Following Hann's derivation [2] but avoiding the incorrect identities in equations (34) and (35), the auto-correlation function of the photomultiplier current, $R(\tau)$, is given by

$$\begin{aligned} R(\tau) \;=\; & A e^{-4\beta^2 u_0^2 \tau^2} e^{-q} \{ \left(1 + cos(D u_0 \tau) J_0(D u'_m)\right) \sum_{m=0}^{\infty} \epsilon_m (-1)^m I_m(q) I_{2m}(p) \\ & + 2\cos(D u_0 \tau) \sum_{m=1}^{\infty} J_{2m}(D u'_m) \sum_{n=0}^{\infty} \frac{\epsilon_n}{2} (-1)^{m+n} I_n(q) [I_{2(n+m)}(p) + I_{2(n-m)}(p)] \\ & - 2\sin(D u_0 \tau) \sum_{m=0}^{\infty} J_{2m+1}(D u'_m) \sum_{n=0}^{\infty} (-1)^{m+n} I_n(q) [I_{2n+(2m+1)}(p) + (1 - \epsilon_n) I_{2n-(2m+1)}(p)] \} \end{aligned} \tag{2}$$

where $p = 2\beta^2 u_o \tau u'_m$, $q = 2\beta^2 u'^2_m$ and $u'_m = \frac{2u_m}{w_m} \sin(\frac{w_m \tau}{2})$ are introduced as abbreviations. β is defined as $\frac{\cos\theta}{d}$ and D is a factor relating the Doppler frequency to the particle speed, and equal to

$$D = \frac{4\pi \sin\theta}{\lambda} \tag{3}$$

ϵ_n is the *Neumann factor* and is defined such that

$$\epsilon_n = \begin{cases} 1 & \text{n} = 0 \\ 2 & \text{n} > 0 \end{cases} \tag{4}$$

where n is an integer. J and I represent the Bessel function and modified Bessel function respectively. A is a constant whose value depends on, among other factors, the density and scattering ability of the tracer particles. Equation 2 differs slightly from the expression presented by Hann, but it was found that when actually plotted as graphs, the two equations were indistinguishable when both the acoustic and the mean velocity amplitudes were small.

2.1 ACOUSTIC FLOW

When there is no mean flow present, $u_0 = 0$, the equation reduces to the expression obtained by Sharpe [3],

$$R(\tau) = e^{-q}\left((1 + J_0(Du'_m)).I_0(q) + 2\sum_{m=1}^{\infty} J_{2m}(Du'_m)I_m(q)\right) \tag{5}$$

if the constant term is omitted. If q is small, corresponding to the acoustic velocity amplitude u_m being considerably less than $\frac{\omega_m \Delta x}{8}$, then equation 5 simplifies further to

$$R(\tau) = (1 + J_0(Du'_m)) = \left(1 + J_0\left(\frac{2Du_m}{\omega_m}\sin\left(\frac{\omega_m \tau}{2}\right)\right)\right) \tag{6}$$

Equation 6 contains a Bessel function with a sinusoidal argument, which is a frequency-modulated signal consisting of a single, prominent frequency component accompanied by side-bands whose relative amplitudes depend on the acoustic velocity.

The first minimum of the zero-order Bessel function occurs when its argument equals 3.832. Since this argument is $\frac{2Du_m}{\omega_m}\sin\left(\frac{\omega_m \tau}{2}\right)$, the acoustic velocity amplitude can be determined by evaluating

$$u_m = \frac{3.832\omega_m}{2D\sin\left(\frac{\omega_m \tau}{2}\right)} \tag{7}$$

2.2 CONSTANT MEAN FLOW VELOCITY

When the acoustic velocity amplitude equals zero, but the mean flow u_0 is non-zero, equation 2 simplifies to

$$R(\tau) = e^{\frac{-\beta^2 u_0^2 \tau^2}{2}}(1 + \cos(Du_0\tau)) \tag{8}$$

which is the damped cosine function predicted by Greated and Duranni [4]. This curve will have peaks whenever the cosinusoidal argument, $Du_0\tau$, equals $2m\pi$, where m is an integer. The mean flow velocity can therefore be determined from the peak separation according to:

$$u_0 = \frac{2\pi}{D\tau_{sep}} \tag{9}$$

where τ_{sep} is the time between peaks.

2.3 INTERPRETATION OF THE ACF

Although equations 7 and 9 provide a means of evaluating the acoustic and mean flow velocities respectively, they are effectively using only one or two points from the ACF- none of the other data points contribute to the calculation. This approach is therefore rather wasteful of the available data.

Another method is to take the Fourier Transform (FT) of the ACF. The FT of a monochromatic cosine wave is simply a delta function, so it is straightforward to extract the velocity if there is a mean flow alone, and no acoustic field. The FT of a Bessel Function with a linear argument is also known, so the acoustic velocity can be obtained if the approximation $\sin\left(\frac{\omega_m \tau}{2}\right) = \frac{\omega_m \tau}{2}$ is valid, that is, for small times. At longer times and higher frequencies, the sinusoidal argument contributes to sidebands in frequency space which in principle could be used to obtain the acoustic velocity amplitude. However, it is not clear whether it is even possible to calculate the FT of equation 6, let alone equation 2. Furthermore, the ACF data set contained a maximum of only 200 points, resulting in limited frequency resolution.

A more effective method is to generate a data set using the relevant equation above, using guesses for u_m and u_0. The sum of the square of the difference between a generated point and the point obtained from the measured ACF provides a measure of the discrepancy. A range of values for u_m and u_0 are attempted, and the values for which the discrepancy defined above is a minimum are then the best estimates for the flow velocities. Although this method of curve-fitting is rather inefficient, it does use more of the measured ACF values and can therefore be expected to be more reliable.

3. RESULTS

Photon correlation results from two experimental set-ups were obtained by MacGillivray [1]: of a standing wave within a closed, glass tube, and of the “free field” in front of a loudspeaker. A Brookhaven BI-9000AT Digital Autocorrelation board performed the autocorrelation of the signal from the photomultiplier. This board had a total of 200 channels, separated in time by $\Delta\tau$, in which values of the discrete ACF were stored. Since it is the actual number of photons which is being correlated, the values of the ACF are rather large, typically 10^9.

This provides a further difficulty: in order to obtain a close fit, the vertical scale of the measured data and the fitted data should be the same. The vertical range of equations 6 and 8 are obvious, so the raw ACF data can be appropriately normalised. The modified Bessel function tends to infinity as its argument increases, so the ranges of equations 2 and 5 are not clear. In attempting to fit these equations to the data, the vertical scale factor is effectively another unknown parameter. This will occur when the acoustic velocity amplitude is significant and/or a mean flow is also present.

3.1 STANDING WAVE RESULTS

A glass tube with one end attached to a loud-speaker and the other end closed was positioned such that the measurement volume was located at the tube’s axis of symmetry.

The effect of the intensity of the sound field and its frequency were investigated. A graph of the ACF against time for a fixed frequency and various pressure amplitudes is shown in figure 2(I). The frequency was 660Hz, a resonance of the tube, and the measured pressures were (a) 1.0Pa, (b) 4.0Pa, (c) 7.0Pa and (d) 10.0Pa. Figure 2(II) shows the effect of frequency on the ACF. A fixed pressure of 5.0Pa was applied for sound frequencies of (a) 660Hz, (b) 1135Hz, (c) 1570Hz and (d) 2000Hz, all of which correspond approximately to resonant frequencies of the tube. All measurements were taken at a velocity antinode. The sample time of the correlator board was $\Delta\tau = 5\mu s$ and the number of channels was 200.

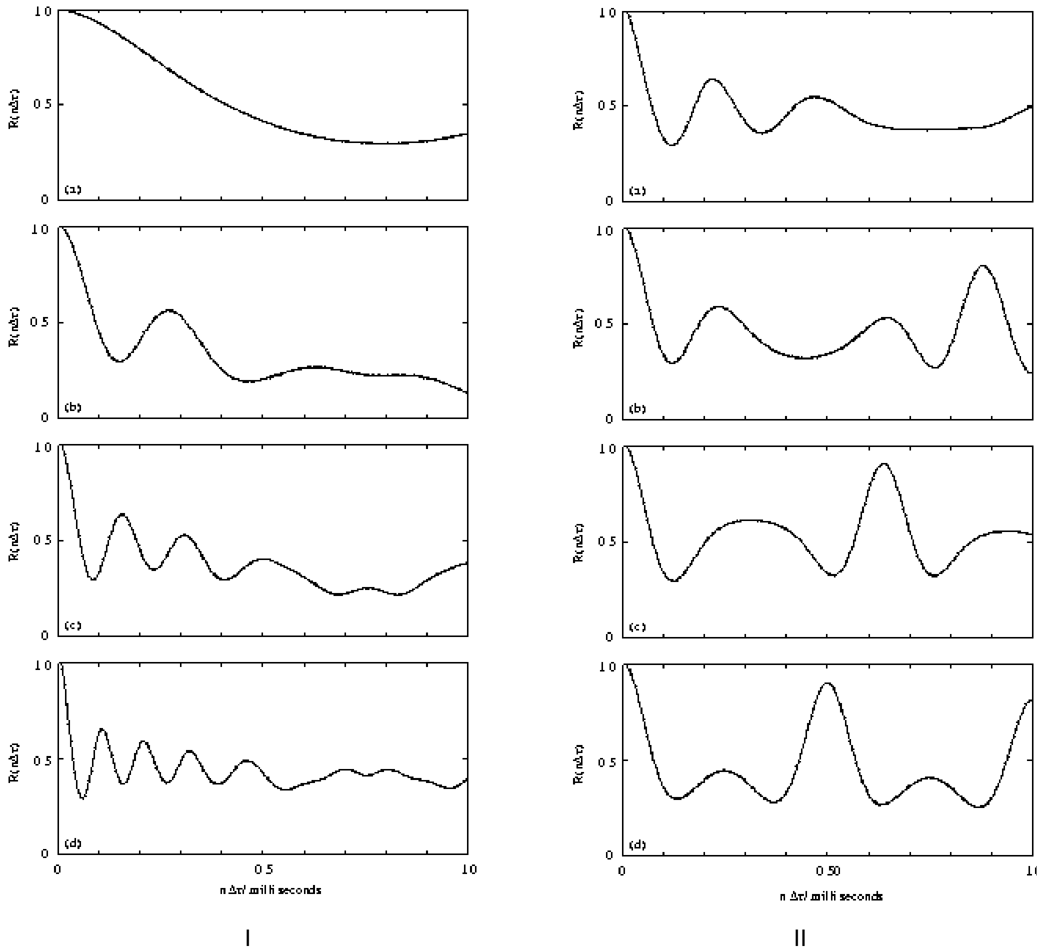

Figure 2: AutoCorrelation Functions showing effect of pressure (I) and acoustic frequency (II)

From figure 2(I) it can be seen that as the pressure amplitude at the rigid end was increased, the time until the first minimum in the ACF decreased. Combining this result with equation 7 suggests that u_m increases with increasing pressure amplitude.

Figure 2(II) shows that the time to the first minimum is independent of frequency. This is predicted by equation 7 if the small angle sine approximation is valid. All of the curves in 2 (I) and (II) contain some damping. This could be accounted for by the presence of a small mean flow. However additional damping factors, not included in equation 2, can also arise from slight misalignments in the laser beams, and a difference in their respective intensities.

MacGillivray attempted to fit a curve of the form

$$R(\tau) = Ae^{-4\beta^2 u_0^2 \tau^2} e^{-q} \left(1 + cos(Du_0\tau)J_0(Du'_m)\right) \tag{10}$$

to the measured data. Equation 10 is a simplified version of 2: it assumes that both an acoustic and a mean flow velocity may be present, but that both are small. The ACF of figure 2(I)(d) and some theoretical fits are shown in figure 3.

By comparing the 20 points of the measured ACF around the first minimum to the values obtained using equation 10, a best estimate of $17.1 \pm 0.1 mms^{-1}$ was found for the acoustic velocity. Theoretical fits were generated with this value of u_m and a range of values for u_0, from 0 to $1 mms^{-1}$. It can be seen that as u_0 increases, the beating interaction between the cosine and the Bessel term becomes more pronounced. The best estimate for the mean flow velocity was $0.6 mms^{-1}$.

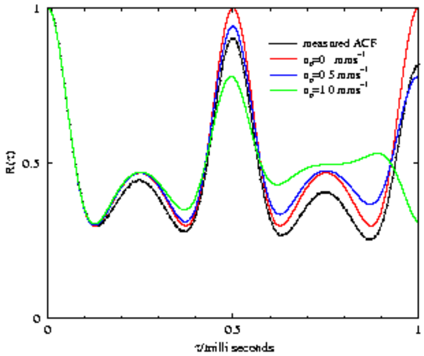

Figure 3: Measured ACF with theoretical fits

Although the theoretical and experimental curves agree well around the first minimum in the ACF, there is an increasing discrepancy with increasing time. Further work will be required to ascertain whether this is due to the terms neglected in equation 10 or misalignments in the experimental setup.

The effectiveness of the photon correlation technique was established by using the LDA measurements to calculate the pressure amplitude, which could be compared directly with measurements from a probe microphone. The pressure values in the frequency range 660-2kHz were found to agree to within 0.25dB.

Experiments were also conducted in the "free field", measuring the velocity amplitude u_m as a function of distance from a loud-speaker. Although the measured ACFs had the qualitative form predicted by theory, the discrepancy with the results obtained from microphone measurements was greater than the calculated uncertainty. This could be due to acoustic reflections, air currents and variations in seeding concentration.

4. CONCLUSIONS

The technique of photon correlation was used to obtain measurements of acoustic fields. A theoretical expression for the form of the auto-correlation function was presented, which assumed that the fluid motion consisted of an acoustic field superimposed on a mean flow. Simplified versions of this equation were least-square-fitted to measured auto-correlation functions which allowed the velocity amplitudes to be determined. Measurements obtained in a standing wave were used to calculate a pressure amplitude which was in good agreement with measurements from a probe microphone. Measurements in the free-field were less accurate but showed that the photon correlation method has the potential to measure flow velocities without the need for calibration.

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