Characterising Modal Interactions in an Ultrasonic Cutting System

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ABSTRACT High power ultrasonic systems are used in industrial applications such as welding and cutting. Common reliability problems, such as noise and component failures, often stem from participation of untuned modes in the driving mode due to nonlinear behaviour. A preliminary theoretical investigation to reduce the system nonlinearities is presented for the case of two Duffing oscillators. Experimental application of the theory to an ultrasonic cutting system demonstrates the validity of manipulating the nonlinear characteristics via 'Nonlinear Cancellation Coupling'. Further, modal interactions in the form of combination resonances are characterised experimentally.

INTRODUCTION

Nonlinear behaviour, to some extent, is present in mechanical and structural systems due to inherent sources of nonlinearities existing even in simple systems. Nonlinearities are responsible for a variety of effects which are absent in linear systems, such as the jump phenomenon, natural frequency shifting, combination resonances, frequency modulations and chaotic motions [1-3]. Many systems excited in vibration, which exhibit linear behaviour at low levels of excitation, become nonlinear at higher levels of excitation. High power ultrasonic systems, designed to resonate in a tuned mode of vibration at a low ultrasonic frequency, typify such systems and are prone to exhibit adverse responses during operation that result from nonlinearities [4]. Nonlinear behaviour of ultrasonic systems is responsible for energy leakage into non-tuned modes leading to an uncontrollable process performance often accompanied by noise and component failures [5].

The application of ultrasonics to manufacturing processes has been an established area of research for several decades. However, many systems, particularly in continuous operation, suffer from component failures and noise. Consequently, the investigation of a method focused on the reduction of nonlinearities is now vital to provide a new boost in the design of more reliable ultrasonic systems.

In this paper, a preliminary theorectical investigation to reduce these undesirable effects has been carried out in the context of 'Nonlinear Cancellation Coupling' of a two degree of freedom system. Here, a second system of an opposite nonlinear characteristic is added to the original nonlinear system. It might seem that this would increase the nonlinearities, but the idea is to 'cancel out' the nonlinearities in the original system. Customisation of this new theory to an ultrasonic cutting device is under investigation. The results of experimental investigations prove that linearisation of an ultrasonic system is possible, thus confirming the theoretical predictions in a general sense.

NONLINEAR CHARACTERISTICS OF AN ULTRASONIC TRANSDUCER

The nonlinear behaviour of ultrasonic systems is strongly attributable to the piezoelectric transducer behaviour [6], since the nonlinear threshold of its constituent ceramics is reached at low excitation. The nonlinear characteristics of an ultrasonic transducer are experimentally analysed below.

Experimental Analysis of Ultrasonic Transducer

Figure 1 shows the experimental setup for a swept-sine excitation of an ultrasonic transducer. The unit under investigation was a commercial 35 kHz longitudinal mode transducer. The transducer was driven by a function generator connected to a power amplifier. The vibration response of the transducer was measured by a 3D laser vibrometer, allowing both in-plane and out-of-plane responses to be characterised. A multi-channel data acquisition analyser connected to a portable computer enabled calculation of frequency response functions (FRFs) and identification of system responses via signal processing software.

Figure 2 shows the frequency response curves for the longitudinal mode of the transducer at two excitation levels. The curves were obtained by sweeping the excitation frequency forwards and backwards. A slow sweep rate was adopted in order to record only steady transducer responses at each frequency increment. The response curve obtained under low excitation (5 V) appeared to be linear. When the same sweep was performed at a higher excitation level (30 V), the jump phenomenon and a natural frequency shift, typical of a nonlinear system, were clearly identified. The curve shows a softening characteristic and an instability region typical of nonlinear systems.

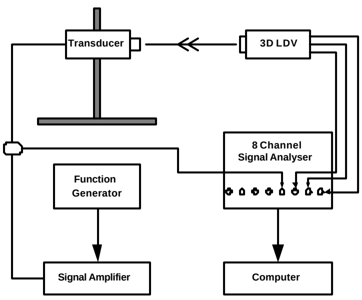


Figure 1 : Schematic diagram of the experimental set-up

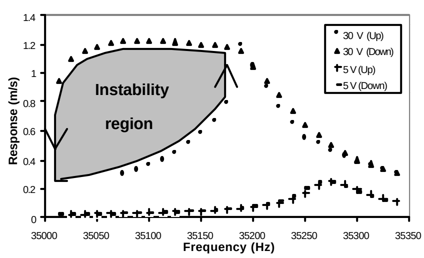


Figure 2 : Frequency response curves of the transducer longitudinal mode

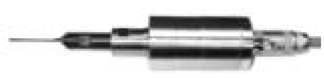


Figure 3 : Transducer-blade ultrasonic cutting system

NONLINEAR CANCELLATION COUPLING

An ultrasonic cutting system, consisting of the transducer and an ultrasonic blade, attached via a threaded stud, is shown in Figure 3. The cutting blade is profiled to provide the required longitudinal amplitude at the cutting edge at 35 kHz. Careful blade design was carried out in order to avoid linear modal participation of close modes in the tuned longitudinal mode response. However, coupling of the longitudinal mode with well-isolated modes through combination resonances is still possible if the system exhibits a nonlinear characteristic. Combination resonances increase significantly with the complexity of systems. Therefore, implementation of ultrasonic technologies consisting of multi-component systems is particularly hampered by nonlinear behaviour. A theoretical and experimental application of a novel method for linearising such coupled nonlinear systems is introduced.

Theoretical Approach (Perturbation Analysis)

Equation of Motion

An analysis is presented for two coupled Duffing-type oscillators (Figure 4) involving linear viscous damping ($c_{1,2}$), and stiffness characteristics modelled by linear and cubic terms ($k_{1,2}$ and $h_{1,2}$ respectively). h_1 is a cubic hardening spring of ($+h_1 x_1^3$) and h_2 is a cubic softening spring of ($-h_2 x_2^3$). A harmonic force is applied to the first sub-system. The governing differential equations for this hypothetical test-bed system are derived,

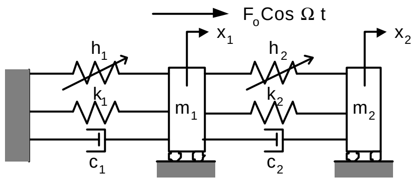


Figure 4 : 2-DOF 'Cancellation Coupling' System

$$m_1\ddot{x}_1 + (c_1 + c_2)\,\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)\,x_1 - k_2 x_2 + h_1 x_1^3 + h_2(x_2 - x_1)^3 \;=\; F_o\ Cos\,\Omega\, t \qquad (1)$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) - h_2(x_2 - x_1)^3 \;=\; 0 \qquad (2)$$

where $x_{1,2} = x_{1,2}(t)$. Introducing dimensionless time, $t = w_{e1}\, t$ where w_{e1} is the undamped linear natural frequency, and using non-dimensional variables such that $\ddot{x}_{1,2} = w_{e1}^2\, x''_{1,2}(t)$ and $\dot{x}_{1,2} = w_{e1}\, x'_{1,2}(t)$, then in dimensionless time, equations (1) and (2) become,

$$x_1'' + e\,\tilde{m}_1 x_1' + e\,\tilde{m}_2 x_1' - e\,\tilde{m}_2 x_2' + (\tilde{a}_1 + e\,\tilde{a}_2)\,x_1 - e\,\tilde{a}_2 x_2 + e\,\tilde{h}_1 x_1^3 + e\,\tilde{h}_2(x_2 - x_1)^3 = e\,\tilde{A}_{o1}\ Cos\,\tilde{W}t \qquad (3)$$

$$x_2'' + e\,\tilde{m}_3 x_2' - e\,\tilde{m}_3 x_1' + \tilde{a}_3 x_2 - \tilde{a}_3 x_1 - e\,\tilde{h}_3(x_2 - x_1)^3 = 0 \qquad (4)$$

where $x_{1,2} = x_{1,2}(t)$ and

$$\tilde{a}_1 = \frac{a_1}{w_{e1}^2}\ ;\ \tilde{a}_2 = \frac{a_2}{e w_{e1}^2}\ ;\ \tilde{a}_3 = \frac{a_3}{w_{e1}^2}\ ;\ \tilde{m}_1 = \frac{m_1}{e w_{e1}}\ ;\ \tilde{m}_2 = \frac{m_2}{e w_{e1}}\ ;$$

$$\tilde{m}_3 = \frac{m_3}{e w_{e1}}\ ;\ \tilde{h}_1 = \frac{h_1}{e w_{e1}^2}\ ;\ \tilde{h}_2 = \frac{h_2}{e w_{e1}^2}\ ;\ \tilde{h}_3 = \frac{h_3}{e w_{e1}^2}\ ;\ \tilde{A}_{o1} = \frac{A_{o1}}{e w_{e1}^2}\ ;\ \tilde{\Omega} = \frac{\Omega}{w_{e1}} \qquad (5)$$

All the terms except the linear stiffness coefficients, $\tilde{a}_1$ and $\tilde{a}_3$ in equations (3) and (4) and also the linear inertia terms are multiplied by the small parameter e, $0 < e << 1$, so that these terms only appear in the higher perturbation equations [7].

Method of Multiple Scales

The classical method of multiple scales [8-9] has been used to solve the equations to second order of approximation. Coefficients of like order of e are grouped and lead to

Order e^0:

$$D_0^2 x_{10} + w_1^2 x_{10} = 0 \qquad (6)$$

$$D_0^2 x_{20} + w_2^2 x_{20} - w_2^2 x_{10} = 0 \qquad (7)$$

Order e:

$$D_0^2 x_{11} + w_1^2 x_{11} = \tilde{A}_{o1}\ Cos\,\tilde{\Omega}\, t - 2D_0 D_1 x_{10} - \tilde{m}_1 D_0 x_{10} - \tilde{m}_2 D_0 x_{10} + \tilde{m}_2 D_0 x_{20}$$
$$- \tilde{a}_2 x_{10} + \tilde{a}_2 x_{20} - \tilde{h}_1 x_{10}^3 - \tilde{h}_2\left(x_{20}^3 + 3x_{10}^2 x_{20} - 3x_{10}x_{20}^2 - x_{10}^3\right) \qquad (8)$$

$$D_0^2 x_{21} + w_2^2 x_{21} = \tilde{m}_3 D_0 x_{10} - 2D_0 D_1 x_{20} - \tilde{m}_3 D_0 x_{20} + w_2^2 x_{11} + \tilde{h}_2\left(x_{20}^3 + 3x_{10}^2 x_{20} - 3x_{10}x_{20}^2 - x_{10}^3\right) \qquad (9)$$

where

$$w_1^2 = \tilde{a}_1 \quad , \quad w_2^2 = \tilde{a}_3 \qquad (10, 11)$$

The harmonic solutions of equations (6) and (7) can be expressed in convenient polar form respectively as

$$x_{10} = A_1(T_1)\, e^{i w_1 T_0} + \overline{A}_1(T_1)\, e^{-i w_1 T_0} \qquad (12)$$

$$x_{20} = A_2(T_1)\, e^{i w_2 T_0} + \overline{A}_2(T_1)\, e^{-i w_2 T_0} + A_3(T_1)\, e^{i w_1 T_0} + \overline{A}_3(T_1)\, e^{-i w_1 T_0} \qquad (13)$$

where $A_3 = G\, A_1(T_1) = \dfrac{w_2^2}{w_2^2 - w_1^2}\, A_1(T_1)$, noting that the overbar denotes complex conjugacy.

The behaviour of this system can be very complicated, especially when the forcing frequency and natural frequencies satisfy certain internal and external resonance conditions. For this system, the following resonances are identified,

External Res. : $\tilde{W} = w_1 + e n$ (14) *Internal Res.* : Superharmonic ; $w_2 \approx \dfrac{1}{3} w_1 + e s_1$ (15)

Subharmonic ; $w_2 \approx 3 w_1 + e s_2$ (16) Primary ; $w_2 \approx w_1 + e s_3$ (17)

en and $es_{1,2,3}$ are conveniently defined as the external and internal detuning parameters respectively. In this paper, only the external and the superharmonic (1 : $\frac{1}{3}$) internal resonances are analysed.

After derivation of the secular terms for the first and second-order expansion, these terms are equated to zero to preserve the uniformity of the expansion. The complex amplitudes A_1 and A_2 can be expressed in polar forms $A_n[T_1] = \frac{1}{2}\, a_n[T_1] e^{i\, b_n [T_1]}$, $\overline{A}_n[T_1] = \frac{1}{2}\, a_n[T_1] e^{-i\, b_n [T_1]}$ where $a_n[T_1]$ and $b_n[T_1]$ are real. A set of modulation equations is derived by separating out the real and imaginary parts of the resulting relations, and imposing steady-state conditions whereby, $a_1' = a_2' = L' = F_1' = 0$ where $L = n T_1 - b_1[T_1]$, $F_1 = s_1 T_1 - b_1[T_1] + 3 b_2[T_1]$. The two solvability equations for superharmonic resonance are then derived as

$$16 e \tilde{A}_{o1}^2 = \left(4 w_1 \left(e\tilde{m}_1 - (\Gamma - 1) e\tilde{m}_2 \right) a_1[T_1] - \frac{16 w_2 e\tilde{h}_2 \left(w_2^2 (e\tilde{m}_2 + e\tilde{m}_3) - w_1^2 e\tilde{m}_3 \right) a_2[T_1]^2}{3(\Gamma - 1)(w_1 + w_2)(w_2 e\tilde{h}_2 + 4(w_2 - w_1) e\tilde{h}_3)\, a_1[T_1]} \right)^2 +$$
$$\frac{\left(\begin{array}{l} 9(\Gamma-1)(w_1+w_2)(4 w_1 e\tilde{h}_3 - w_2 (e\tilde{h}_2 + 4 e\tilde{h}_3))\, a_1[T_1]^2 \left(8 e n w_1 + 4(\Gamma-1) e\tilde{a}_2 - 3 \left(e\tilde{h}_1 + (\Gamma-1)^3 e\tilde{h}_2 \right) a_1[T_1]^2 \right) \\ + 2 e\tilde{h}_2 \left(8 w_2 \left(2 (en - 3 e s_1) \left(w_1^2 - w_2^2 \right) + 3 w_2 e\tilde{a}_2 \right) - 9(\Gamma-1)^2 \left(w_2 (w_2 - 3 w_1) e\tilde{h}_2 + 8 \left(w_1^2 - w_2^2 \right) e\tilde{h}_3 \right) a_1[T_1]^2 \right) a_2[T_1]^2 \\ - 36\, e\tilde{h}_2 \left(w_2^2 (e\tilde{h}_2 + e\tilde{h}_3) - w_1^2 e\tilde{h}_3 \right) a_2[T_1]^4 \end{array} \right)^2}{81(\Gamma-1)^2 (w_1 + w_2)^2 (w_2 e\tilde{h}_2 + 4(w_2 - w_1) e\tilde{h}_3)^2\, a_1[T_1]^2}$$

(18)

$$144 \left(w_1^2 w_2 e\tilde{m}_3 - w_2^3 (e\tilde{m}_2 + e\tilde{m}_3) \right)^2 + \left(\begin{array}{l} -4 w_2 \left(2 (en - 3 e s_1) \left(w_1^2 - w_2^2 \right) + 3 w_2 e\tilde{a}_2 \right) \\ + 18 (\Gamma-1)^2 \left(w_2^2 (e\tilde{h}_2 + e\tilde{h}_3) - w_1^2 e\tilde{h}_3 \right) a_1[T_1]^2 + 9 \left(w_2^2 (e\tilde{h}_2 + e\tilde{h}_3) - w_1^2 e\tilde{h}_3 \right) a_2[T_1]^2 \end{array} \right)^2$$
$$= \frac{81}{16} (\Gamma-1)^2 (w_1 + w_2)^2 (w_2 e\tilde{h}_2 + 4(w_2 - w_1) e\tilde{h}_3)^2\, a_1[T_1]^2\, a_2[T_1]^2$$

(19)

Analytical Results

For the hardening system, the nonlinear effect changes significantly when a further nonlinear sub-system is coupled to it. Figure 5 shows the responses of the system, accordingly to equations (18) and (19), plotted in the frequency domain. As the softening stiffness, h_2 , is decreased, with the rest of the variables constant, it tends to bend the nonlinear response characteristic to the right, to a more linear-like response.

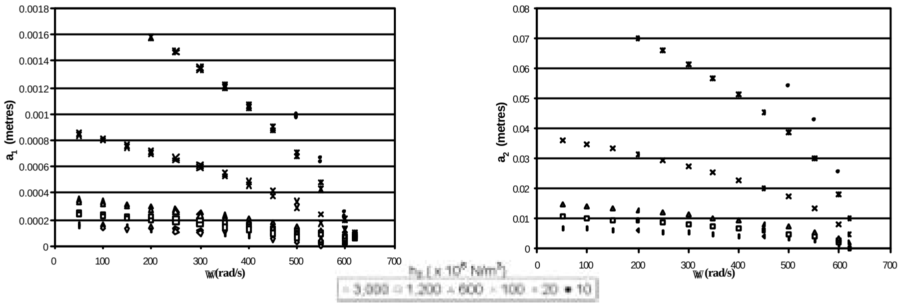


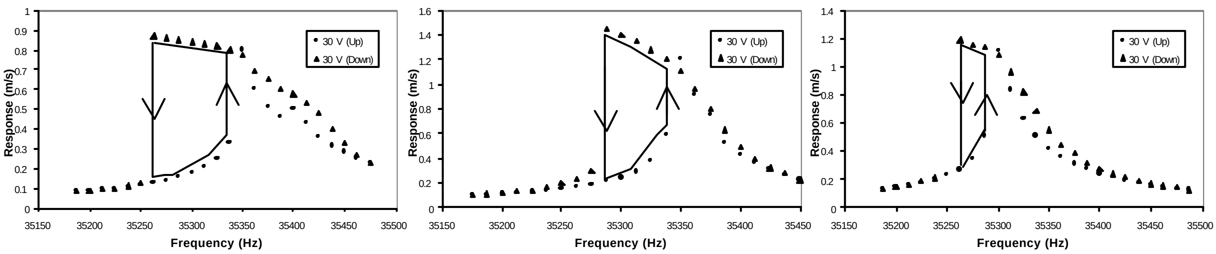
Figure 5 : Frequency response plot for variation of the softening stiffness characteristic, h_2.

m_1=2 kg ; m_2=2 kg ; c_1=c_2=0.05 Ns/m ; k_1=847776 N/m ; k_2=52986 N/m ;

Application of the 'Nonlinear Cancellation Coupling' concept to the ultrasonic cutting system

The transducer showed evidence of a nonlinear softening characteristic typical of a Duffing oscillator, as illustrated in Figure 2. The method of 'Nonlinear Cancellation Coupling' of the 2-DOF system is now applied to the transducer-blade system, in order to manipulate the effects of its nonlinear behaviour.

An experimental forward and backward sine-sweep of the excitation frequency was carried out on this system, using the same set-up as described previously. The transducer excitation level was kept constant at 30 V and the response at the shoulder of the blade was monitored by the 3D LDV. An attachment configuration with the threaded stud fully-screwed into the end of the blade base was first examined. Figure 6a shows a clear nonlinear softening response of the cutting system, but comparatively more linear than the response of the transducer alone. This may be due to the cutting blade having a hardening characteristic, typical of beam-like structures, with the implication that the 'Nonlinear Cancellation Coupling' is valid in principle. According to the theory, opportune variations of one of the nonlinear stiffnesses ($h_{1,2}$) of the Duffing oscillators would improve the linearity of the system. In order to apply those changes to the ultrasonic cutting assembly, different locations of the threaded stud were considered. Attachment configurations with the stud half-screwed into the transducer (Figure 6b) and fully-screwed into the end of the transducer base (Figure 6c) were also investigated. The frequency sweep results demonstrated a more linear behaviour for both stud locations. In particular, Figure 6c revealed a remarkable reduction of the system's nonlinear characteristic. It appeared that variations of local stiffness at the joint location have a significant influence on the system nonlinear characteristics, providing further evidence of the validity of the linearisation method.



(a) : Threaded stud fully-screwed into the end of blade-base

(b) : Threaded stud half-screwed into the end of blade-base

(c) : Threaded stud fully-screwed into the end of transducer-base

Figure 6 : Frequency response curves of the transducer-blade longitudinal mode

COMBINATION RESONANCES IN AN ULTRASONIC CUTTING SYSTEM

Modal interactions in nonlinear systems can arise when the system is harmonically excited in the vicinity of a natural frequency. In particular, if special relationships (combination resonances) between two or more linear modes and the excitation frequency exist, the system response contributes more modes. Effects of combination resonances on ultrasonic systems are high noise level, component fatigue and poor operating performance. Experimental detection of modal interactions in the transducer-blade assembly, characterised by the lowest degree of nonlinearity, was carried out. The modes were identified initially by measuring the FRFs. The out-of-plane and two in-plane FRFs from a point on the blade are presented in Figure 7a. At first the system was driven in the vicinity of the first longitudinal frequency of the cutting blade (35 kHz). The excitation frequency was swept forward and backward over a range of 300 Hz, at 5 V increments of excitation level. As soon as the excitation level reached 30 V, two peaks corresponding to two linear modes appeared in the response spectrum (Figure 7b). These modes, occurring at 16.9 kHz and 18.4 kHz, correspond to the third bending mode of the transducer-blade assembly and the third bending mode of the blade, respectively. The sum of their frequencies being equal to the longitudinal frequency was clear identification of a combination resonance. The contribution of these two modes to the overall response appeared to be marginal due to their low response. Therefore, the performance of the cutting system driven at the first longitudinal mode was not critically affected by the nonlinearly excited modes.

For the sake of completeness, the system was subsequently driven at 43 kHz, a frequency corresponding to the second longitudinal mode of the assembly, which is not the mode the system was designed for. A sweep of the excitation frequency was again performed over a 300 Hz range for increasing excitation increments. Two peaks, occurring at 4 kHz and 39 kHz, corresponding to the first and the sixth bending modes of the system respectively, appeared in the spectrum at an excitation of 20 V (Figure 7c). In this case both of the nonlinearly excited modes were highly responsive, in comparison with the linearly driven mode. The system became very noisy due to the excitation of the audible 4 kHz mode. The excitation level versus frequency plot in Figure 8 shows the instability regions inside which the combination resonances participate within the response. The region of the second combination resonance is wider than the first, and the threshold at which such modal interactions appear is lower.

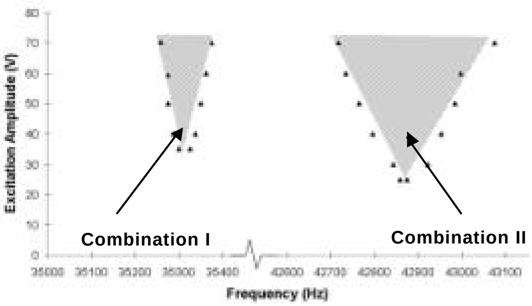


Figure 8 : Stability regions for the transducer-blade ultrasonic cutting system

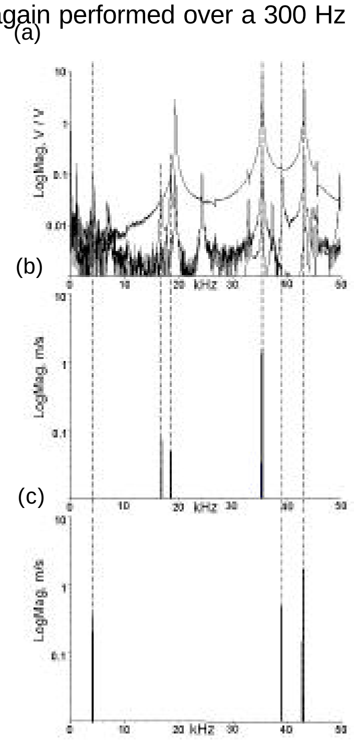


Figure 7 : (a) Out-of-plane and two in-plane FRFs of transducer-blade system ;
(b) Combination resonance I ; (c) Combination resonance II

CONCLUSIONS

Research has shown that nonlinearities in high power ultrasonic systems mainly stem from the transducer behaviour. A novel method for influencing the nonlinear behaviour of a system is proposed for a hypothetical, but relevant, case of two coupled Duffing oscillators. Experimental application of this method to the case of an ultrasonic cutting system proved capable of reducing the effect of the nonlinearities within the system. In particular, the effect of the configuration of the threaded stud connection appeared to be crucial in the control of nonlinearities. In addition, two combination resonances, having a marginal and a stronger effect on the operation response of a cutting system, have been experimentally detected and described.

Applicability of the 'Nonlinear Cancellation Coupling' method to complex multi-component ultrasonic systems, characterised by numerous natural frequencies, if proved, would represent a major advance in high power ultrasonic tool design.

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