B.2

ZARAGOZA '89 ENVIRONMENTAL ACOUSTICS

8º SYMPOSIUM FASE'89
«ACUSTICA AMBIENTAL»
Zaragoza, Abril 1989

SOUND PROPAGATION IN THE TURBULENT ATMOSPHERE A COMPARISON OF APPROXIMATION METHODS

Ralf Große, Volker Mellert, Manfred Schultz-von Glahn, Albert Sill

Universität Oldenburg, Postbox 2503, D-2900 Oldenburg, FRG

1. Introduction

The propagation of noise in the atmosphere is calculated only for average sound velocity profiles until today. To describe fluctuations of the noise level caused by an irregular motion of the air a statistical theory is necessary. Although no such theory exists there are several methods to obtain approximate solutions of the problem. This paper reviews these methods in order to compare their respective validity ranges.

2. Helmholtz equation

Sound propagation in the turbulent atmosphere is described by a linear partial differential equation /1/ which contains the wind and temperature field. Most of the literature deals with the scalar Helmholtz equation, ignoring two components of the wind vector. The one component left is combined with the temperature to an acoustical refractive index. Its relative deviation from the mean refractive index is the random variable introducing the statistics to the Helmholtz equation.

$$\left(\Delta + k^2 \left(1 + \mu(\vec{r})\right)\right) \psi(\vec{r}) = 0 \qquad (1)$$

Δ = *Laplace operator;* k = *wave number;* μ = *refractive index deviation.*

We are looking for the first statistical moments of the wave function to be measured as phase and amplitude fluctuations for a given autocorrelation function of the refractive index deviation.

3. Approximation methods

A principal difficulty in solving equation (1) stems from its stochastical nonlinearity. In spite of its apparent linearity it contains a product of two random variables. Therefore the problem can only be solved approximately. There are two small parameters used for such approximations: the size of the relative refractive index deviations and the ratio of the wave length to the length of a typical inhomogeneity of the medium, described by the correlation length. To compare the results obtained by the various methods described in the following we presume a gaussian autocorrelation function.

$$< \mu(\vec{r})\,\mu(\vec{r}\,') > = < \mu^2 > \; exp\left\{ -\frac{(\vec{r} - \vec{r}\,')^2}{l^2} \right\} \tag{2}$$

l = *correlation length*

3.1 Perturbation-theoretical methods

If slight refractive index deviations lead only to small wave fluctuations the wave function can be expanded in powers of this small parameter. The first order approximation to this series is the Born approximation known from quantum theory.

$$(\Delta + k^2)\; \psi_1(\vec{r}) = k^2 \mu(\vec{r})\, \psi_0(\vec{r}) \tag{3}$$

ψ_0 = *incident (unperturbed) wave;* ψ_1 = *perturbed wave*

A variant of this perturbation method is the Rytov method /1/ in which the logarithm of the wave function is expanded. For the quantities we are interested in, the differences between the two methods are not very significant /2/.

Equation (3), linear also in the stochastical sense, is solved by an integral over its inhomogeneity and the Greens function of the homogeneous equation. The mean value of the perturbed wave function is zero, because the mean of the random variable vanishes. An analytical solution for the second statistical moment only exists for the small wave length limit. On this assumption the Greens function is changed (Fresnel approximation). By the same argumentation the integral can be simplified further /2/. The calculation, using (2), leads to:

$$< |\psi_1|^2 > = |\psi_0|^2\, \alpha z \quad ; \quad \alpha := \sqrt{\pi} < \mu^2 > k^2\, l \tag{4}$$

In reverse order of the approximations used above the Helmholtz equation can be transformed into the eikonal equation of geometrical optics which is solved using the perturbation method /1/. The validity of this result is more restricted than that of the Born approximation. The results are only identical in the limit of small wave lengths. The range of validity of all perturbation methods is restricted to small scattering volumes. With increasing scattering volume the wave fluctuations are increasing too and the convergence of the perturbation series is destroyed.

3.2 Parabolic equation method

For a small wave length - correlation length ratio the Helmholtz equation can be converted into a parabolic form /1/.

$$\left(2ik\frac{\partial}{\partial z} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\mu(\vec{r}) \right)\; \psi(\vec{r}) = 0 \tag{5}$$

This transformation is completely equivalent to the Fresnel approximation and is less restrictive than the transformation into the eikonal equation. Physically it means the assumption of small scattering angles /3/. There are several methods to solve equation (5): statistical moment equations /4/, path integrals /5/, local method of small perturbations /3/. To eliminate the stochastical nonlinearity in (5) all these methods assume that the wave propagation is a Markov process referring to the main propagation direction z. Successive scatterings are assumed to be statistically independent. The justification of this assumption is mainly based on the small wave length compared to the correlation length.

Its physical meaning becomes clear in the "local method of small pertubations" /3/. The scattering volume is divided into layers, that are thicker than the correlation length, but thinner than the validity range of the Born approximation. The existence of such layers requires small refractive index deviations. On this assumption the single scattering events take place in distances larger than the correlation length, and can therefore be considered as uncorrelated. The results for the first two moments are:

$$< \psi > = \psi_0 \; exp\Big\{ -\frac{\alpha}{2} z \Big\} \tag{6}$$

$$< |\psi|^2 > = |\psi_0|^2 \tag{7}$$

The scattered intensity /4/ is:

$$< |\psi|^2 > - | < \psi > |^2 = |\psi_0|^2 \; (1 - exp\{-\alpha z\}) \tag{8}$$

α = saturation coefficient defined in (4)

Using at least the same presumptions as the Born approximation, this result contains the whole perturbation series. It shows the saturation of wave fluctuations as a multiple scattering effect. For short propagation distances it reduces to the Born approximation result (4). For the validity of (8) it is necessary that the wave length is small compared to the correlation length. Moreover, the refractive index deviations have to be small, but not as small as it is necessary for the application of the Born approximation to the whole scattering volume. It is only required that the saturation coefficient alpha appearing in (8) and (4), is small enough so that the mean value of the wave does not change very much over the distance of one correlation length /4/.

3.3 Smoothing method

The atmosphere always contains structures of the same size as the wave length, so that their ratio is of the order of unity. The wave length cannot be assumed to be small. Otherwise the mean value of the wave function does not decrease significantly over the distance of a correlation length. To measure this kind of "smallness", a parameter R (generalized Reynolds number) is defined by:

$$R := \sqrt{< \mu^2 >} \, k^2 \, l^2 \tag{9}$$

The difference between the wave function and its mean value, the fluctuating part of the wave, is expanded in powers of R to obtain an equation for the mean value only /6/. Since the mean value varies much less than the wave itself, this expansion is called "smoothing method". The first order smoothing approximation leads to an integro-differential equation.

$$(\Delta + k^2) < \psi(\vec{r}) > = -k^4 \int d^3 \, \vec{r}' \; G(\vec{r}, \vec{r}') < \mu(\vec{r}) \mu(\vec{r}') > < \psi(\vec{r}') > \tag{10}$$

G = Greens function of free propagation

The same expression can be obtained by the use of Feynman diagrams /6/. It is solved by Fourier transformation /7/.

$$< \psi > = \psi_0 \; exp\Big\{ -\frac{\beta}{2} z \Big\} \quad ; \quad \beta := \alpha \, (1 - exp\{-k^2 l^2\}) \tag{11}$$

Multiple scattering effects are included in equation (10), since it is based on an infinite partial sum of the Born series. So the saturation effect is expressed by the result (11). In the small wave length limit it converges to the result of the parabolic equation method (6). An integro-differential equation for the second moment of the wave function can be developed by the same method /6/. Analytical solutions of this equation exist only for the case that the waves are either short or long compared to the correlation length.

4. Conclusions

Two small parameters are used in methods for the approximative solution of the stochastic, scalar Helmholtz equation: the relative refractive index deviation and the wave length - correlation length ratio. They are applied in three different ways: for perturbation expansions (3.1), as a justification of the parabolic equation and the Markov-assumption (3.2), and combined to the generalized Reynolds number for the iteration of the fluctuating part of the wave (3.3). These approximation methods were described here in a sequence of increasing validity ranges. Because of its relative independence of wave length restrictions, the smoothing method is most suitable to the problem. Otherwise a combination of methods could be useful to cope with the special difficulties arising from inhomogeneous and anisotropic turbulent fields. For example, an anisotropic refractive index profile can be treated by the method of geometric optics deterministically (ray tracing), and the turbulence is described by a parabolic equation related to the rays. To describe the noise propagation in the turbulent atmosphere more generally, an approximation method without any wave length restriction should be applied to a vector Helmholtz equation with a refractive index tensor containing the whole wind vector.

The authors wish to thank Prof. Karl Haubold for the valuable discussions during the whole work.

Literature

/1/ *Tatarskii, V.I., The effects of the turbulent atmosphere on wave propagation* Jerusalem 1971

/2/ *Clifford, S.F., The classical theory of wave propagation in a turbulent medium* in: *Strohbehn, J.W., Laser beam propagation in the atmosphere*, Berlin 1978

/3/ *Strohbehn, J.W., Modern theories in the propagation of optical waves in a turbulent medium* in: *Strohbehn, J.W. Laser beam propagation in the atmosphere* Berlin 1978

/4/ *Uscinski, B.I., The elements of wave propagation in random media* New York 1977

/5/ *Dashen, R., Path integrals for waves in random media*, J.Math.Phys. (5), 1979, 894–920.

/6/ *Frisch, U., Wave propagation in random media*, in: *Bharucha-Reid, A.T. Probabilistic methods in applied mathmatics*, New York 1968

/7/ *Soczkiewicz, E; Chivers, R.C., Propagation of acoustic waves in random media using the parabolic equation and Greens function methods*, Acustica (65), 1988, 149–155.