On spherical wave scattering by randomly rough interfaces between elastic media

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Introduction

In spite of the fact that plane waves are most frequently assumed in scattering studies, due to the fact that it is possible to decompose a general acoustical signal into plane wave components [1, 2], spherical waves form a more realistic case when comparison is performed with experimental values obtained using piston type transducers for insonification of a rough scattering surface.

Theoretical derivations of spherical wave scattering with far field approximations are similar to that of plane wave scattering [3]. Numerical simulations have been performed in order to show the difference between the scattered fields when a plane wave and a spherical wave is used, respectively [4]. Experiments of wave scattering were performed using the surface of sand in a box immersed in a water tank [5]. The sand surface was originally assumed to be smooth, but the experimental results were different from what should be expected for wave reflection from smooth surfaces. The experiments described in Ref. [5] were later re-done under the assumption that the interface between water and sand possesses some kind (e.g. a random type) of roughness.

The incident spherical wave

The pressure field of a spherical wave in a free space may be expressed by:

$$p(R) = P_0 \exp(ikR) / R, \quad R = (x^2 + y^2 + z^2)^{1/2} \qquad (1)$$

where P_0 is the pressure amplitude given by $P_0 = [W\rho c/2p]^{1/2}$, W is the power output of the sound source [3], ρ is the density of water, c is the sound velocity in water, and k is the wave number.

The spherical wave is incident on a randomly rough interface between two elastic media. Due to the limited width of the rough interface in the experimental studies some edge effects may have to be taken into account [6]. owever, a considerable reduction of the edge effects may be obtained by exploitation of the directivity qualities of the acoustical wave emitted by a piston source. A Gaussian type directivity function frequently used in 2D theoretical studies may be expressed by:

$$D(x_0,y_0) = D_0 \exp(-x_0^2 / L_x^2 - y_0^2 / L_y^2) \qquad (2)$$

where D_0 is a constant (set to unity at the center of the beam pattern), (x_0, y_0) represents a point on the surface, L_x and L_y are characteristic lenghts of the directivity function in x and y directions, respectively.

The wave scattering functions

After the directivity function and the spherical wave have been chosen, the incident pressure wave is now given by:

$$p(R) = P_0 D \exp(ikR) / R \qquad (3)$$

Starting from Helmholtz theorem

$$p(R) = 1/4\pi \int [G(\partial p / \partial n)|_s - p|_s (\partial G / \partial n)]ds \qquad (4)$$

where G is the full space Green's function:

$$G(R) = \exp(ikR) / R \qquad (5)$$

and where n is a normal drawn toward the source, S is the rough surface area, and $p|_s$ and $\partial p/\partial n|_s$ are the values of p(R) and its normal derivatives on the surface. The origin of the coordinate system is in the center of the illuminated area (see Fig. 1). By using the Kirchhoff approximation and the far field approximation, omitting the detailed derivations, the scattered pressure field can be expressed by [3, 6]:

$$p(R) \sim [ikP0\ f(\theta_1,\theta_2,\theta_3) V(\theta_1,\theta_2,\theta_3) e^{ik(R_1+R_2)} / 2\pi R_1 R_2] \iint_{-\infty}^{\infty} D_0 e^{2i(\alpha x_0 + \beta y_0 + \gamma\zeta)} dx_0 dy_0 \qquad (6)$$

where $f(\theta_1,\theta_2,\theta_3)$ is the angular function in scattering. For scattering in the specular direction ($\theta_2 = \theta_1$, $\theta_3 = 0$), $f(\theta_1,\theta_2,\theta_3) = \cos\theta_1$, and for backscattering ($\theta_2 = \theta_1$, $\theta_3 = 0$), $f(\theta_1,\theta_2,\theta_3) = 1/\cos\theta_1$, θ_3 is the side scattering angle. $V(\theta_1,\theta_2,\theta_3)$ is the reflection coefficient at the interface (being a function of all the three angles [1] and assumed to be constant over the whole surface). R_1 and R_2 are shown in Fig. 1, α, β and γ are x-, y- and z-components of the sum of the incident and scattered wavenumber vectors, respectively. And ζ is the local surface height deviating from the mean surface (see Fig. 1). The integration should be taken over the whole surface, but in numerical calculations, only a finite area can be integrated.

By assuming a randomly rough surface, a statistical technique should be used in evaluating Eq. [6], and the scattered field can be considered to consist of two parts: A coherent and an incoherent part.

While the coherent part of the scattered field is regular and in phase and, therefore, a decomposition and a recomposition (or FFT) technique can be used for this part, the incoherent part of the scattered field is irregular and out of phase, and statistical techniques may most appropriately be used for this field part.

By averaging Eq. (6) over the random surface roughness, the coherent scattering can be obtained as:

$$<p>_\zeta \sim p_0 V <W(\zeta) exp(i\gamma\zeta)> \quad (7)$$

where p_0 is the expression by setting $\zeta = 0$, or the scattering from a perfectly reflecting smooth surface with same dimensions as the rough surface. The second part of Eq. (7) represents the coherent scattering coefficient by considering both the surface roughness and the medium elasticities. The corresponding incoherently scattered field is given by:

$$<s^2>_\zeta \sim [k^2 P_0^2 f^2 V^2 L_x L_y / 2\pi R_1^2 R_2^2] \iint_{-\infty}^{\infty} D_0 e^{i(\alpha x_0+\beta y_0)} \{e^{-4\gamma^2\sigma^2[1 - C(x_0,y_0)]} - e^{4\gamma^2\sigma^2}\} dx_0 dy_0 \quad (8)$$

where a Gaussian spectrum and correlation function for the surface roughness has been assumed, σ is the rms height, and $C(x_0,y_0)$ is the correlation function.

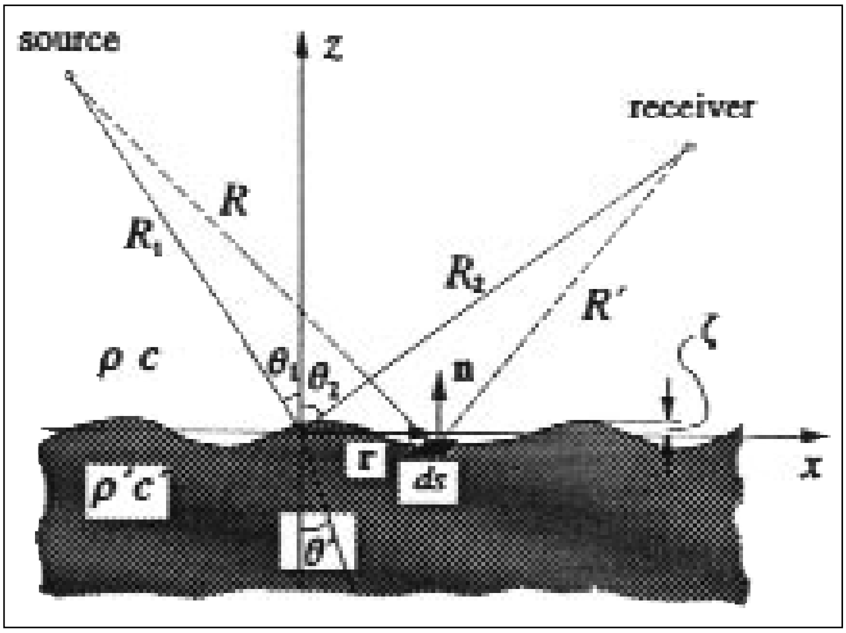

Fig nº 1.- Geometry of wave scattering from rough interfaces between elastic media.

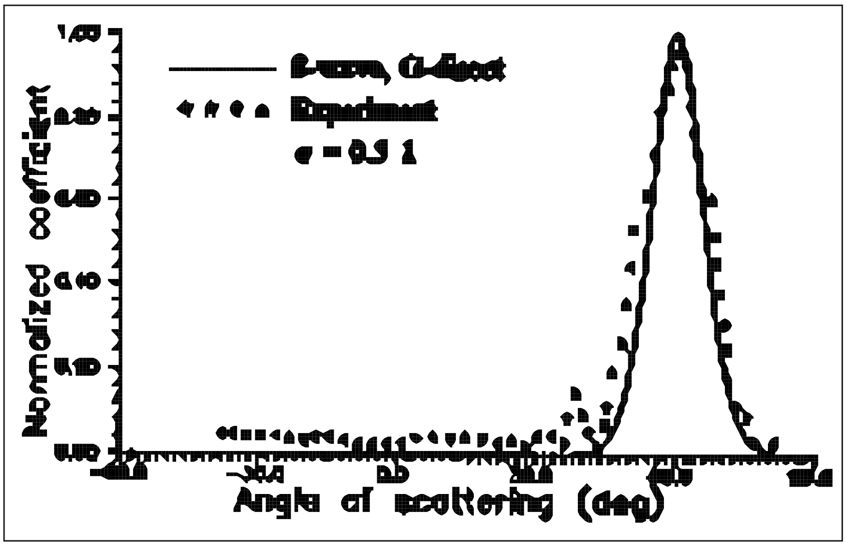

Fig nº 2.- Comparisons of experimental and numeric results.

Numerical simulations

By using a Gaussian spectrum for the roughness distribution on the rough surface, the coherent scattering can be given in a very simple expression from Eq. (7), as:

$$<p>_\zeta \sim p_0 V exp(-2\gamma^2\sigma^2)> \quad (9)$$

Eq. (9) shows the influence of the surface roughness (σ) on the coherently scattered field, it indicates that the coherent scattering is only affected by the rms surface height. Eq. (8) determines the influence of the surface roughness on the incoherent scattering, which as shown is affected by both the surface roughness height and the correlation between different surface points.

Numerical simulation studies based on Eq. (8) and Eq. (9) have been performed for various correlations lengths and rms surface roughness heights and the results have been compared with experimental data.

Experimental set-up

The scattering experiments from rough surfaces were conducted in an indoor water-tank of which the dimensions are 2 meters in width and depth, and 3 meters in length. The rough surface was formed by a water/sand interface. A box, of which the dimensions were 20 cms in length and width and 10 cm in height, was used to contain the sand. The sand box and a 180Ï steel arch were fixed in the water-tank. Two transducers were mounted on the arch in a way to permit them to move smoothly along the arch such that different angles of incidence and scattering could be studied experimentally.

Discussions

Gated sine-wave trains were used in the experiments and the signal amplitudes of the incident and the scattered fields were measured. By assuming a σ of the surface roughness to be 1/2 wavelength (center frequency used was 500 kHz), the correlation length of the rough surface to be 10 wavelengths, and by using a Gaussian directivity function, the numerical results for forward scattering are shown in Figure 2 for an angle of incidence of 60º. As expected from physical reasonings the maximum of the scattered field is found in the specular direction. A minor discrepancy between the experimental and the numerical results is found when the angle of incidence deviates from the specular direction. The experimental patterns are slightly broader and a distribution of some sound energy in other directions is found.

Conclusions

Scattering of spherical waves from a randomly rough (Gaussian roughness distribution) interface between two elastic media (water and sand) has been studied numerically and experimentally. It is found that the rms height of the roughness distribution is the most important parameter influencing the scattered field. The numerical results show that more acoustical energy is propagated in the specular direction for smaller roughness heights, while increasing roughness heights will lead to an increased incoherent scattering part.

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